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Browder's Theorem For Totally Posinormal Operators

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ABSTRACT

In our study we consider a higher class of Hilbert space operators, Posinormal operators introduced by C.Rhaly(1992).The purpose of this paper is to prove that if A is a Totally Posinormal operator such that $\sigma(A-\lambda I)_{|M} = 0 \implies (A-\lambda I)_{|M} = 0$ for every $M \in Lat(A)$ and satisfies property(ab),then A satisfies Browder's theorem and generalized Browder's theorem. We shall also prove that, if N is a nilpotent operator such that $AN = NA$,then Browder's theorem holds for $A + N$.

Keywords: Totally posinormal Operators, property(ab), SVEP, Browder's theorems

1. INTRODUCTION

Throughout this paper,$B(H)$ denotes the space of all bounded linear operators acting on an infinite dimensional Hilbert space H into itself.

Definition 1.1[1]An operator$A\in B(H)$ is said to be posinormal if there exists a positive operator$P \geq 0 \in B(H)$such that$AA^* = A^*PA$. Equivalently,$A \in B(H)$is posinormal if there exists a co-isometry$V^* \in B(H)$ and positiveoperator $P \in B(H)$ such that $A = A^*PV^*A$,$A$is called a posinormal operator with interrupter $P \in B(H)$.

Remark 1.2 From [1], we have the inclusion;

Hyponormal⊆ M-hyponormal⊆Dominant ⊆Posinormal.

Definition 1.3[2] A posinormal operator A is said to be totally posinormal abbreviated TP if there exists a positive operator P such that $A \in B(H)\colon |(A-\lambda I)^*|^2 = |P^{\frac{1}{2}}(A-\lambda I)|^2$ for all$\lambda \in \mathbb{C}$.

Definition 1.4[2]A is conditionally totally posinormal abbreviated CTP if to each $\lambda \in \mathbb{C}$ there corresponds a positive operator $P_\lambda$such that $A \in B(H)\colon |(A-\lambda I)^*|^2 = |P_\lambda^{\frac{1}{2}}(A-\lambda I)|^2$ for all $\lambda \in \mathbb{C}$.

2. LITERATURE REVIEW

Several authors have shown in [5], [14],[4], that Weyl's theorems holds for Hyponormal

operators. It was shown in [13] that M-hyponormal operators satisfy Weyl's theorem. Weyl's theorem is not satisfied by dominant operators that are equivalent to $TP$and CTP operators and therefore it is important to establish necessary and sufficient conditions that implies Weyl's theorem for TP and CTP operators.Weyl's theorem for totally posinormal operators was introduced in [2],the authors considered the class of operators on$H$such that if $\sigma(A) = \{0\}$implies that $A = 0$ denoted by$Q$, it was proved that if A is totally posinornal and $A_{|M} \in Q$for every$M \in lat(A)$then;

i) A is isoloid i.e $iso\sigma(A) \subset \sigma_p(A)$;
ii) Asatisfies Weyl's theorem;
iii) If N is a nilpotent operator commuting with$A$,then $A + N$ satisfies Weyl's theorem[2];
iv) If $F$is a finite rank operator commuting with$A$,then $A + F$ satisfies Weyl's theorem[2];
v) If $f \in \mathcal{H}(\sigma(A)$,then Weyl's theorem holds for$f(A)$, where $\mathcal{H}(\sigma(A)$is the set of functions analytic in the neighborhood of spectrum of A.[2].

In Theorem 9 [2], conditions under which a totally posinormal is isoloid is established and using this result, Mecheri[15] proved if $A \in B(H)$ is conditionally totally posinormal operator such that$(A - \lambda I)_{|M} \in \mathcal{Q}$for every$M \in Lat(A)$,then $f(A)$ satisfies the generalized Weyl's theorem for every $f \in H\big(\sigma(A)\big)$which implies that Weyl's theorem holds for $f(A)$. Recently several authors have established variants of Weyl's theorem and Browder's theorem as useful tools for establishing the theorems for classes of operators in Hilbert spaces.

In [7] the authors introduced property(b) and property(gb) which are variants of a-Weyl's theorem and generalized a-Weyl's theorem respectively. As a continuation, the properties were extended to properties(ab); (gab); (aw); (gaw) in [8] as variants of Browder's theorem and generalized Browder's theorem.

3. NOTATION AND TERMINOLOGY

For $A \in B(H)$ let A^*; $R(A)$; $N(A)$; $\rho(A)$; $\sigma(A)$; $\sigma_p(A)$; $\sigma_{ap}(A)$ denote respectively adjoint, range, kernel, resolvent set, spectrum, point spectrum and approximate point spectrum.

Definition 3.1[11] Let $A \in B(H)$,the ascent is defined as the smallest positive integer

$p = p(A)$ such that$N(A)^p = N(A)^{p+1}$, the descent of A is the smallest integer $q = q(A)$such that $R(A)^q = R(A)^{q+1}$.

Definition 3.2[3] Let $\alpha(A) < \infty$ and $\beta(A) < \infty$ be the nullityand the deficiency of A defined by $\alpha(A) = dimN(A)$ and $\beta(A) = \dim R(A)^{\perp}$and index of A denoted by;

$ind(A) = \alpha(A) - \beta(A)$.

Definition 3.3[15] If $R(A)$ is closed and $\alpha(A) < 1 \; or \; \beta(A) < 1$ then A is called an upper

semi-Fredholm operator or a lower semi-Fredholm operator, we denote the class of upper semi-Fredholm operators in H by;$\Phi_+(H) := \{A \in B(H) : \alpha(A) < 1$ and $R(A)$ is closed $\}$while theclass of lower semi-Fredholm operators is denoted by ;$\Phi_-(H) := \{A \in B(H) : \beta(A) < 1\}$.

The class of allFredholm operators is defined as$\Phi_(H) := \Phi_+ H) \cup \Phi_-(H)$.

The essential spectrum(or the Fredholm spectrum) is defined by;

$$\sigma_e(A) = \{\lambda \in \mathbb{C}: \lambda I - A is not Fredholm\}.$$

It is clear that the essential spectrum is a non-empty closed subset of$\mathbb{C}$.

The essential approximate point and essential surjective spectrum are defined respectively as;

$\omega_a(A) = \{\lambda \in C: A - \lambda I \text{ is not upper} - Fredholm\};$

$\sigma_{\delta e}(A) = \{\lambda \in C: A - \lambda I \text{ is not lower} - Fredholm\}.$

For operators on a Hilbert space$\sigma_e(A)$ coincides with$\omega_a(A)$and $\sigma_{\delta e}(A)$.

Definition 3.4[12] Let $\mathcal{W}(H)$ denote the class of Fredholm operators of indexzero known as Weyl operators and the Weyl spectrum denoted by$\omega(A) = \{\lambda \in C: \lambda I - A \text{ is not } Weyl\}$.

Remark 3.5

i) We say that Weyl's theorem holds for A if $\sigma(A)\backslash\omega(A) = \pi_{\circ\circ}(A)$where $\pi_{\circ\circ}(A)$=$\{\lambda \in iso\sigma(A): 0 < \alpha(A - \lambda I) < 0\}$is the set of isolated points of $\sigma(A)$ which are eigenvalues of finite multiplicity.[11]

ii) We say that a-Weyl's theorem holds for $A \in B(H)$ if $\sigma_{ap}(A)\backslash\omega(A) = \pi_{\circ\circ}^{a}(A)$,where $\pi_{\circ\circ}^{a}(A)$denotes the set of isolated points of$\sigma_{ap}(A)$ which are eigenvalues of finite multiplicity.[11]

It has been shown in [13] that;a-Weyl's theorem$\Longrightarrow$Weyl'stheorem .

Definition 3.5[11] An operator $A \in B(H)$ is upper semi-Browder if it is upper semi-Fredholm and has finite ascent,this class of operators is denoted by;

$B_+ := \{A \in \Phi_+(H) : p(A) < 1\}$.Similarly, $A$is lower semi-Browder if it is lower semi-Fredholm and has finite descent, this class is denoted by$B_- := \{A \in \Phi_-(H) : q(A) < 1\}$.An operator $A$is Browder if it is both lower and upper semi-Browder. Equivalently this means that A is Fredholm and has finite both ascent anddescent. The class of all lower and upper Browder operators (also known as RieszSchauder operators)is denoted by;

$B = B_+ \cap B_- = \{A \in \Phi(H): p(A), q(A) < \infty\}.$

Definition 3.6[11] The set$\sigma_b(A) = \{\lambda \in \mathbb{C}: \lambda I - A is not Browder\}$is the Browder spectrum of A .The Browder essential approximate point spectrum of $A$is defined by;

$\sigma_{ab}(A) = \cap \{\sigma_{ab}(A + K): AK = KA and K \in B(H)\}.$

Evidently $\sigma_e(A) \subseteq \omega(A) \subseteq \sigma_b(A) \subseteq acc\sigma(A) \cup \sigma_e(A) \subseteq \sigma(A)$.

Remark 3.7

i) We say that Browder's theorem holds if$\sigma(A)\backslash\omega(A) = p_{\circ\circ}(A)$,where$p_{\circ\circ}(A) = \sigma(A)\backslash\sigma_b(A)$. [15]

ii) We say a-Browder's theorem holds for $A \in B(H)$if$\sigma_{ap}(A)\backslash\sigma_{ab}(A) = \pi_{\circ}^{a}(A)$, where $\pi_{\circ}^{a}(A)$ is the set of all left poles of finite rank.[15]

Several authors have shown that for any $A \in B(H)$ the following implications hold;

a-Weyl's theorem$\Longrightarrow$ Weyl's theorem $\Longrightarrow$ Browder's theorem

a-Weyl's theorem $\Longrightarrow$ a-Browder's theorem $\Longrightarrow$Browder's theorem

Definition 3.8[7] For $A \in B(H)$ and a nonnegative integer n define $A_{[n]}$to be the

restriction of A to $R(A^n)$ viewed as a map from $R(A^n)$ into itself.Iffor some integer n ,$A_{[n]}$is a Fredholm operator, then A is called a B-Fredholm operator. An operator $A \in B(H)$ is said to be a B-Weyl operator if it is a B-Fredholm operator of index zero.

Definition 3.9[7] The semi-B-Fredholm spectrum of A is the set $\sigma_{SBF}(A) = \{\lambda \in C: \lambda I - A \ is\ not\ a\ semi\ B - Fredholm\ operator\ \}$.

The B-Weyl spectrum is defined by;

$\sigma_{BW}(T) = \{\lambda \in C: \lambda I - A\ \ is\ not\ a\ \ B - Weyl\ operator\}$.

Remark 3.10[7] We say that Generalized Weyl's theorem holds for A if $\sigma(A)\backslash\sigma_{BW}(A) = E(A)$ where $E(A)$ is the of all isolated eigenvalues of A and the generalized Browder's theorem holds for A if $\sigma(A)\backslash\sigma_{BW}(A) = \pi(A)$where $\pi(A)$ is the set of all poles of$A$. The semi-essential approximate point spectrum is defined by; $\sigma_{SBF_{+}^{-}}(A) = \{\lambda \in \mathbb{C}: \lambda I - A \notin SBF_{+}^{-}(H)\ \}$where$SBF_{+}^{-}(H)$isthe class of all $A \in SBF + (H)$ such that $ind(A) \leq 0$.

Remark 3.11 The concept of Drazininvertibility which plays an impor-tant role for class of B-Fredholm operators,this concept was originally considered by Drazin,[9] where Drazin invertible elements are called Pseudo-invertible elements. In the case of $A \in B(H)$ following Proposition 6[16],A is Drazin invertible if and only if it has finite ascent and descent.

Definition 3.12[16] The Drazin spectrum is defined by;$\sigma_D(A) = \{\lambda \in \mathbb{C}: \lambda I - A\ is\ not\ Drazin\ invertible\}$and $A \in B(H)$ is said to be left Drazin invertible if;$A \in LD(H) = \{A \in B(H): p(A) < 1\ and\ R(A)^{p(A)+1}\ \ is\ closed\}$.

We say that $A \in B(H)$ obeys the generalized$a$-Browder's theorem if

$\sigma_{SBF_{+}^{-}}(A) = \sigma_{ap}(A)\backslash\pi^{a}(A)$ where$\pi^{a}(A)$ is the setof all left poles of$A$.

The authors in [6] have shown that;

generalized Browder's theorem $\Longrightarrow$Browder's theorem generalized a-Browder's theorem$\Longrightarrow$ a-Browder's theorem

Definition 3.13 An operator $A \in B(H)$have Single value extension property(SVEP)

if for any analytic function $f : D \to H$ with $(\lambda I - A)f(\lambda) \equiv 0$,it results $f(\lambda) \equiv 0$.

Remark 3.14[11] In local spectral theory, the local spectral subspace is defined by;$\chi_A(\Omega) := \{x \in H: \sigma_A(x) \subseteq \Omega\}$ and the quasi-nilpotent part of $\lambda I - A$is defined by;

$$H_{\circ}(\lambda I - A) := \left\{x \in H: \lim_{n\to\infty} \|(\lambda I - A)^n\|^{\frac{1}{n}}\right\} := \chi_A(\mathbb{C}\setminus\{\lambda\})$$

While $\chi_A(\mathbb{C}\setminus\{\lambda\})$coincides with the analytic core defined as the set $K(\lambda I - A)$for all $a \in H$ such that $\exists\, c > 0$ and a sequence(a_n) in$H$ for which$(\lambda I - A)a_1 = a, (\lambda I - A)a_{n+1} = a_n$and $\|a_n\| \leq c^n\|a\|$for all $\in \mathbb{N}$.

Definition 3.13[3]An operator $A \in B(H)$ is said tohave Bishop's property $(\beta\,)$ if $f_n(\lambda)$ is an analytic vector valued function on some open set D such that $(\lambda I - A)f_n(\lambda) \to 0$uniformlyon each compact set $K \subset D$,then $f_n(\lambda) \to 0$ as $n \to \infty$,again locally uniformly on K.

Definition 3.14[11]A bounded operator $A \in B(H)$, is said to have the Dunford property(C),if the analytic subspace $\chi_A(\Omega)$is closed for every closed subset $\Omega \subseteq \mathbb{C}$.

It has been shown in [3]that;Property(β) $\Longrightarrow$Property(C) $\Longrightarrow$SVEP.

4. MAIN RESULTS

Definition 4.1[8] An operator $A \in B(H)$ is said to possess property(ab) if $\sigma(A)\setminus\omega(A) = \pi_{\circ}^{a}(A)$and is said to possess property (gab) if$\sigma(A)\setminus\sigma_{BW}(A) = \pi^a(A)$.

Proposition 4.2 If $A \in B(H)$ possesses property(gab),then A possesses property(ab)

Proof

Suppose A possesses property(gab),then if$\lambda \in \sigma(A)\setminus\omega(A)$, then $\lambda \in \sigma(A)\setminus\sigma_{BW}(A) = \pi^a(A)$since $\sigma_{BW}(A) \subseteq \omega(A)$, and $A - \lambda I \in \Phi_+(H)$ it followsthat $\alpha(A - \lambda I) < 1$ and therefore by definition 3.12, A is Drazin invertible hence $\lambda \in \pi_{\circ}^{a}(A)$.

Conversely suppose $\lambda \in \pi_{\circ}^{a}(A\,)$, then λ is a left pole of $\sigma_{ap}(A)$which implies that$\alpha(A - \lambda I)$and $p(A - \lambda I)$ are finite, thus following theorem 3.8[11],A has SVEP and by Theorem 2.13[10]$\lambda \in \sigma(A)\setminus\omega(A)$.

Lemma 4.3 If A is totally posinormal(conditionally totally posinormal) then $N(A - \lambda I) = N(A - \lambda I)^2$for any $\lambda \in \mathbb{C}$ and A has SVEP.

Proof

Since $N(A-\lambda I) \subseteq N(A-\lambda I)^2$,we show that $N(A-\lambda I)^2 \subseteq N(A-\lambda I)$.Suppose A is totallly posinormal then by definition it follows that $(A-\lambda I)(A-\lambda I)^* = (A-\lambda I)P(A-\lambda I)^*$ for some positiveoperator P and $N(A-\lambda I) \subseteq N(A-\lambda I)^*$,therefore let$x \in N(A-\lambda I)^2$,then$(A-\lambda I)x \in N(A-\lambda I)^*$ and consequently since$A-\lambda I$is bounded,$0 = \|(A-\lambda I)^*(A-\lambda I)x\|\|x\| \geq \|(A-\lambda I)x\|^2$which implies that$x \in N(A-\lambda I)$.Thus Ahas a finite ascent and by Theorem 3.8[11]T has SVEP

Remark 4.4 By Theorem 2.40[11] it can be shown that if $A \in CTP$or$\in TP$, then $f(A) \in \mathcal{H}(\sigma(A))$ has SVEP.

Theorem 4.5 Let $A \in B(H)$ be a totally posinormal operator. Then$A$ satisfies property(ab)

Proof

Suppose $\lambda \in \sigma(A)\backslash\omega(A)$, then since by Theorem 13,[2] A satisfies Weyl'stheorem it follows that $\lambda \in \pi_{\circ\circ}(A)$.Following equivalent conditions in Theorem 2.13[10] $\exists\, d \geq 1$ for which$H_\circ(\lambda I - A) = N(A-\lambda I)^d$,which implies that$A \in \mathcal{P}(H)$ thus every$\lambda \in \pi_{\circ\circ}(A)$ is an isolated point of the resolvent of Ai.e.$\lambda \in \pi_\circ(A) \subseteq \pi_\circ^a(A)$,hence $\sigma(A)\backslash\omega(A) \subseteq \pi_\circ^a(A)$.

Conversely without loss of generality suppose$0 \in \pi_\circ^a(A)$, since$0 \in iso\sigma(A)$

and every totally posinormal operator is isoloid it follows that $\lambda \in \sigma_{ap}(A)$.Considering the Reisz projection$P = \int (A-\lambda I)^{-1} d\lambda_{|\lambda|=\eta>0}$corresponding to 0;$M := PH$ is an

invariant subspace for A and $\sigma(A_{|M}) = \{0\}$ implies that $A_{|M} = 0$therefore $A \in \mathcal{Q}$ thus

thus $M = N(A) \neq \{0\}$and $S = A_{|PH}$ invertible, therefore let$A = \begin{bmatrix} 0 & 0 \\ 0 & S \end{bmatrix}$ on $H = PH \oplus PH^\perp$.

Let $K = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$on $H = PH \oplus PH^\perp$.

Since$\dim(PH) < \infty$and $A+K$ is invertible,A is Weyl, i.e.$0 \notin \omega(A)$thus$\pi_\circ^a(A) \subseteq \sigma(A)\backslash\omega(A)$.

Corollary 4.6 Let $A \in B(H)$ be totally posinormal and $A_{|M} \in \mathcal{Q}$for every $M \in Lat(A)$.

If $f \in \mathcal{H}(\sigma(A))$,then $f(A)$satisfies property(ab)

Proof

Following remark 4.4, $f(A)$ has SVEP and from Theorem 16[4]and Corollary2.14[8]that $f(A)$ satisfiesWeyl's theorem and therefore in asimilar proof to Theorem 3.5 it can be shown that $f(A)$satisfiesproperty(ab).

Theorem 4.7 Let $A \in B(H)$ be totally posinormal and $A_{|M} \in \mathcal{Q}$for every $M \in Lat(A)$,then Asatisfies Browder's theorem.

Proof

Suppose$p_{00}(A)$, then $\alpha(A-\lambda I) < 1$,$p(A-\lambda I) < 1$ and$R(A-\lambda I)^{p(A-\lambda I)+1}$ is closed which implies that $A-\lambda I \in LD(H)$ and $\lambda \in \pi_0^a(A) = \sigma(A)\backslash\omega(A)$since A possessesproperty(ab).

Conversely suppose $\lambda \in \sigma(A)\backslash\omega(A)$, $\lambda \in \pi_{00}(A)$since A,satisfies condition C it follows that$p_{00}(A) \subseteq \pi_{00}(A)$hence$\sigma(A)\backslash\omega(A) \subseteq p_{00}(A)$.

Corollary 4.8 Let $A \in B(H)$ be totally posinormal and $A_{|M} \in Q$for every $M \in Lat(A)$.If A satisfies property(gab) then generalized Browder's theorem holds for A.

Proof

Suppose that Asatisfies property(gab) then$\sigma(A)\backslash\sigma_{BW}(A) = \pi^a(A)$ and since A satisfies Weyl's theorem and$\sigma_{BW}(A) \subseteq \omega(A)$,then$\pi_{00}(A) = \sigma(A)\backslash\omega(A) \subseteq \sigma(A)\backslash\sigma_{BW}(A)$,thus$\pi_{00}(A) \subseteq \pi^a(A) \subseteq \pi(A)$and hence$\sigma(A)\backslash\omega(A) \subseteq \pi(A)$.

Conversely, suppose$\lambda \in \pi(A)$then $p(A-\lambda I) < \infty$and the space$R(A-\lambda I)^{p(A-\lambda I)+1}$ is closed,it follows that A is Drazin invertibleand therefore$\lambda \in \pi^a(A)$ since $A$is isoloid.Following theorem 2.2[10],A possesses property(gab) if it possesses property(ab)hence $\lambda \in \pi^a(A) = \sigma(A)\backslash\sigma_{BW}(A)$.

Theorem 4.9 Let $A \in B(H)$ and$N \in B(H)$ be a nilpotent operator such that $AN = NA$, then A satisfies property(ab) if and only if $A + N$ satisfies property(ab).

Proof

Suppose A satisfies property(ab),then $\lambda \in \pi_0^a(A) = \sigma(A)\backslash\omega(A)$so $A-\lambda I$ is a Weyl operator.By definition of the spectrum of an operator under perturbation by a nilpotent operator, we have$\sigma(A+N) = \sigma(A); \omega(A+N) = \omega(A)$ this follows from lemma 2[4] and it follows that the operator $A+N-\lambda I$ is Weyl, then following Corollary2.6[8], Browder's theorem holds for $A+N$which implies that $\lambda \in p_{00}(A+N) = \sigma(A+N)\backslash\omega(A+N)$and $p_{00}(A+N) = \pi_0^a(A+N)$.

Conversely suppose $A + N$ satisfies property(ab),then$A = (A+N) - N$ satisfies property(ab).

Corollary 4.10 Let $A \in B(H)$ be totally posinormal and $A_{|M} \in Q$for every $M \in Lat(A)$.IfN is nilpotent operator such that $AN = NA$,then $A+N$satisfies property$(ab)$and satisfies Browder's theorem.

Proof

The proof follows from Theorem 4.9 and Theorem 2.4[8].

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