BCM 103: Introduction to Business Mathematics

Developer:	Dr. Yabesh Ombwori Kongo (PhD)
Programme:	Bachelor of Commerce

OPEN UNIVERSITY OF KENYA

COURSE PURPOSE

This course provides leaners with a comprehensive foundation in business mathematics in business disciplines and equipping them with the skills needed to succeed in various business environments.

EXPECTED LEARNING OUTCOMES

By the end of this course, the learner should be able to:

1. Describe Concepts and Principles of Business Mathematics
2. Determine Basic Algebraic Operations and Calculus to solve optimization problems
3. Analyze and evaluate Quantitative Methods for Business Decision-Making
4. Evaluate analytical Skills through Mathematical Reasoning to real-world business problems

COURSE CONTENT

Introduction to Business Mathematics: Definition of basic mathematical concepts, Fundamental mathematical tools in various business scenarios. **Basic Algebra and Arithmetic:** Real Numbers and Their Properties, Operations with Algebraic Expressions, Linear Equations and Inequalities, Applications in Business (e.g., break-even analysis). **Functions and Graphs:** Definition and Types of Functions (Linear, Quadratic, Exponential, Logarithmic), Graphing Functions (Linear, Quadratic, Exponential and Logarithmic Functions); Business Applications of functions (cost, revenue, and profit functions). **Matrix Algebra:** Introduction to Matrices, Matrix Operations, Applications of Matrices in Business. **Optimization Problems in Business:** Introduction to basic Optimization Problems; Linear Programming; Formulation of Linear Programming Problems, Graphical Solution Method, Applications in Resource Allocation and Optimization maximising profit and minimising cost. **Mathematics of Finance**: Simple and Compound Interest, Present and Future Value (money) Calculations, Annuities and Perpetuities, Loan Amortization and Investment Appraisal. **Calculus for Business:** Limits and Continuity; Basic concepts of Differentiation and Integration; Differentiation Rules, procedure and Applications of Derivatives in Business (consumer and producer surplus, Marginal Analysis, Optimization); Integration Rules, procedure and application in Business (Total Cost and Revenue). **Decision Analysis:** Decision-Making under Uncertainty, Expected Value and Decision Trees, Risk Analysis. **Statistics for Business:** Descriptive Statistics; Measures of Central Tendency and Dispersion.

LIST OF MODULES

1. Introduction to Business Mathematics
2. Basic Algebra and Arithmetic
3. Functions and Graphs
4. Matrix Algebra
5. Optimization Problems in Business
6. Mathematics of Finance
7. Calculus for Business
8. Decision Analysis
9. Statistics for Business

TABLE OF CONTENT

i	Maximum Modules for each course is set 12

MODULE 1: INTRODUCTION TO BUSINESS MATHEMATICS

INSTRUCTIONAL HOURS: 3

MODULE OVERVIEW

In this module, you will learn about the foundational aspects of business mathematics, focusing on the essential mathematical concepts that underpin various business operations. We will cover key definitions and introduce fundamental mathematical tools that are vital in analyzing and solving real-world business problems. Through practical examples and applications, you will explore how these mathematical concepts are applied in different business scenarios, such as financial analysis, optimization, and decision-making. By the end of this module, you will have a solid understanding of how to use basic mathematical principles to approach and solve common business challenges, equipping you with the skills necessary to make informed and effective business decisions.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Describe key mathematical concepts essential for business operations.
2. Interpret the application of fundamental mathematical tools in various business scenarios.
3. Apply mathematical techniques in real-world business problems to make informed decisions.
4. Evaluate the effectiveness of mathematical models and methods in solving business challenges and optimizing business processes.

LEARNING ACTIVITIES/TASK LIST

1. Complete personal introduction video
2. Listen to the module introductory message
3. Review power presentation PDF
4. Masterly exercise 1.1
5. Watch video
6. Participate in online discussion forum
7. Participate in end of module assessment

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning

READING MATERIAL 1

(Notes, presentation, an article, a book chapter, case study, video, please

	add assessment…)

Activity 1: Personal introduction video

You are required to do a 2 minutes introduction video and share in the LMS. Kindly remember to mention your expectation at the end of this module in your video.

Activity 2: Course Introductory Message

You are required to listen to the prerecorded course introductory challenge message on the role of mathematics in business.

Upon completion;

I. Reflect on the diverse areas where mathematical techniques are applied in business. Consider how these techniques influence decision-making, optimize operations, and contribute to achieving business objectives.
II. Think about specific examples from finance, marketing, operations, and strategic planning where mathematics plays a crucial role.
III. Finally, contemplate how developing your mathematical skills can enhance your ability to solve real-world business problems and drive success in your future career.

Challenge Message

Imagine you as a business oriented person running a business without clear goals, effective strategies, or the ability to make informed decisions. The role of business is to create value, drive innovation, and meet the needs of customers while ensuring profitability and sustainability.

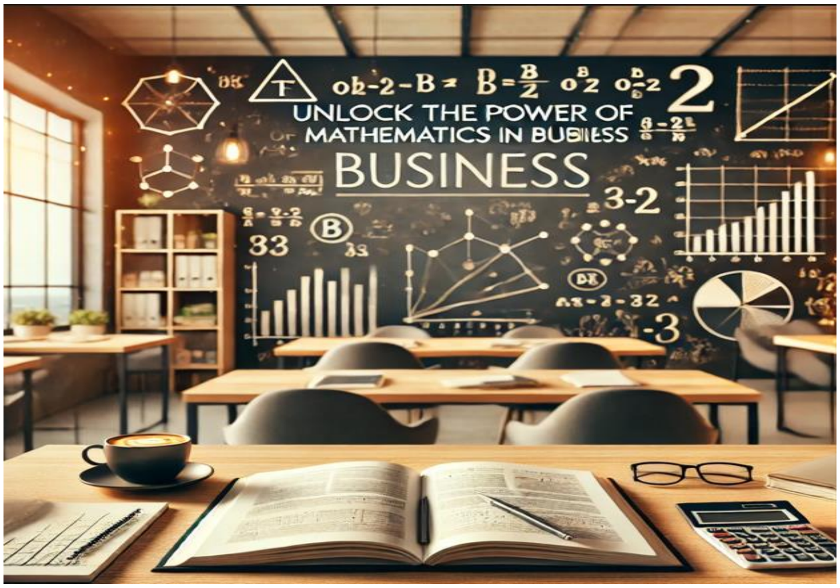
However, achieving these objectives requires more than just intuition or experience; it demands precision, analysis, and strategic thinking.

This is where mathematics becomes indispensable.

Mathematics provides the tools to quantify business activities, optimize resources, and predict future trends. Whether it's analyzing financial performance, optimizing production processes, or making data-driven decisions, mathematical techniques are at the core of solving complex business challenges.

By mastering these techniques, you'll be equipped to not only understand the numbers behind business decisions but also to drive growth and success in an increasingly competitive landscape.

Are you ready to unlock the power of mathematics in business? Welcome to this module as we unlock the power of mathematics in business environments.



Activity 3: You are required to review power point presentation notes on introduction to Business Mathematics.

Upon completion; undertake the quiz or questions labeled **(Task 1)**

Introduction to Business Mathematics

What is Business?

Business refers to the organized efforts and activities of individuals or groups to produce, sell, and exchange goods and services for profit. The primary goal of any business is to create value for its customers while generating profit for the owners or shareholders.

Businesses can vary in size, from small local shops to large multinational corporations, and they operate in various industries such as manufacturing, retail, finance, healthcare, and technology.

Examples of Business Activities:

1. **Manufacturing**: Producing goods like cars, electronics, or clothing.

2. **Retail**: Selling products directly to consumers, such as in supermarkets or online stores.
3. **Services**: Offering intangible goods like consulting, education, or healthcare.

Illustration 1: Imagine a local bakery that produces and sells bread and pastries. The business involves purchasing raw materials (flour, sugar), baking the products, and then selling them to customers. The bakery must manage its finances, set prices, market its products, and ensure customer satisfaction to stay profitable.

What is Mathematics?

Mathematics on the other hand is the study of numbers, quantities, shapes, and patterns, and it involves using various techniques to solve problems and make decisions.

Mathematics is a broad field that includes areas such as arithmetic, algebra, geometry, calculus, and statistics. It provides tools and methods that help us understand and analyze the world around us.

Examples of Mathematical Concepts:

1. **Arithmetic**: Basic calculations such as addition, subtraction, multiplication, and division. For instance, calculating the total cost of goods purchased.
2. **Algebra**: Solving equations and understanding relationships between variables.

Illustration 2: finding the value of x in an equation like;

$$2x + 5 = 11$$

3. **Geometry**: Studying shapes, sizes, and properties of space. An example would be calculating the area of a rectangular plot of land.
4. **Calculus**: Analyzing changes and motion, such as determining the rate at which a quantity changes over time.
5. **Statistics**: Collecting, analyzing, and interpreting data. For example, calculating the average income of a group of people.

Illustration 3: Consider a scenario where you need to calculate the total cost of a shopping cart filled with various items. You would use arithmetic to sum up the prices of each item, and if there's a discount applied to the total, you might use algebra to determine the final amount you need to pay.

Business Mathematics

Business Mathematics is the application of mathematical methods and techniques to solve problems in a business context. It involves using mathematics to perform calculations, analyze data, optimize operations, and make informed decisions.

Business Mathematics is essential for various functions within a business, including finance, accounting, marketing, operations, and strategic planning.

Key Areas of Business Mathematics

1. **Financial Mathematics**:

Illustration 3: Calculating interest on a loan. If a business takes out a loan of Kshs10,000 at an annual interest rate of 5%, it can use simple interest or compound interest formulas to determine how much it will owe over time.

Businesses use financial mathematics to assess the cost of borrowing, determine pricing strategies, and evaluate investment opportunities.

2. **Algebra in Business**:

Illustration 4: Break-even analysis: This involves finding the point at which total revenue equals total costs. Using algebraic equations, a business can determine how many units of a product it needs to sell to cover its costs.

Algebra helps businesses analyze profit margins, forecast sales, and create financial models.

3. **Statistics and Data Analysis**:

Illustration 5: Market research: Businesses collect data on customer preferences, market trends, and competitors. By applying statistical methods, they can interpret this data to make informed marketing and product development decisions.

Statistics is crucial for quality control, customer satisfaction surveys, and risk management.

4. **Optimization and Operations Research**:

Illustration 6: Supply chain management: Businesses must optimize their inventory levels, transportation routes, and production schedules to minimize costs and maximize efficiency. Techniques like linear programming are used to solve these optimization problems.

Optimization techniques help businesses streamline operations, reduce waste, and improve resource allocation.

5. **Calculus in Business**:

Illustration 7: Marginal analysis: Businesses use calculus to determine the marginal cost or revenue of producing one more unit of a product. This information is vital for making decisions about production levels and pricing.

Calculus is therefore used in cost-benefit analysis, economic modeling, and understanding the dynamics of supply and demand.

6. **Decision Analysis**

Decision-Making under Uncertainty: Techniques for making decisions when outcomes are uncertain.

Expected Value and Decision Trees: Tools for evaluating and visualizing decision options.

Risk Analysis: Assessing and managing risks associated with business decisions.

Illustrations of Business Mathematics in Action

1. **Pricing Strategy**:

   A company selling smartphones needs to determine the optimal price for its product. By analyzing the relationship between price, demand, and profit using algebraic functions, the company can set a price that maximizes its profit.

2. **Loan Amortization**:

   When a business takes out a loan, it must repay it over time. Using financial mathematics, the business can calculate its monthly payments and the total interest paid over the life of the loan.

3. **Inventory Management**:

   A retailer must manage its inventory to avoid stock outs and overstocking. By applying statistical models, the retailer can predict demand and optimize its inventory levels.

4. **Investment Appraisal**:

   Before investing in a new project, a business must evaluate its potential return. Using techniques like net present value (NPV) and internal rate of return (IRR), the business can assess whether the investment is worthwhile.

Importance of Business Mathematics

1. Simplification: At times students believe that mathematics makes business and economics concepts more complex. Algebraic notation, which is essentially a form of shorthand, can, however, make certain concepts much clearer to understand than if they were set out in words. It can also save a great deal of time and effort in writing out tedious verbal explanations.

   Illustration 8: The relationship between the quantity of bananas consumers wish to buy and the price of bananas might be expressed as: "the quantity of bananas demanded in a given time period is 1,200 kg when price is zero and then decreases by 10 kg for every 1KES rise in the price of a kilo of bananas. Much simpler, however, this can be mathematically expressed as:

   $Q=1200-10P$ Where Q is the quantity of bananas demanded in kilograms and P is the price in KES per kilogram of bananas.

   This is a very simple example. The relationships between economic variables can be much more complex and mathematical formulation then becomes the only feasible method for dealing with the analysis.

2. Decision-Making: Provides quantitative tools for making informed decisions based on data analysis.
3. Financial Management: Helps in budgeting, forecasting, and financial planning.
4. Optimization: Assists in resource allocation, cost minimization, and profit maximization.
5. Data Interpretation: Facilitates the understanding and interpretation of market trends, performance metrics, and business statistics.

Limitations of Business Mathematics

While business mathematics is a powerful tool for decision-making and analysis, it does have certain limitations. Here are some key limitations:

1. Assumptions and Simplifications

- **Limitation:** Mathematical models often rely on simplifying assumptions that may not fully capture real-world complexities.
- **Example:** Linear programming models assume linear relationships between variables. In reality, relationships may be non-linear or subject to external factors, leading to less accurate results.

2. Data Quality and Availability

- **Limitation:** The accuracy of mathematical analysis depends on the quality and reliability of the data used.
- **Example:** Forecasting models require accurate historical data. If the data is incomplete or inaccurate, the predictions generated by the model may be misleading.

3. Dynamic and Uncertain Environments

- **Limitation:** Business environments are dynamic and subject to uncertainty, which can affect the relevance of mathematical models.
- **Example:** Economic downturns, market disruptions, or changes in regulations can impact the assumptions used in financial models, making past calculations less relevant.

4. Over-Reliance on Quantitative Analysis

- **Limitation:** Focusing solely on quantitative analysis may overlook qualitative factors that are crucial for decision-making.
- **Example:** A mathematical model might suggest investing in a high-return project, but factors such as company culture, employee morale, or customer satisfaction might be ignored, which could affect the overall success of the project.

5. Complexity of Advanced Techniques

- **Limitation:** Advanced mathematical techniques can be complex and require specialized knowledge, which may not be readily available within an organization.
- **Example:** Techniques like stochastic modeling or advanced statistical analysis require a deep understanding of mathematics and statistics, which may be challenging for non-experts to interpret and apply effectively.

6. Ethical and Strategic Considerations

- **Limitation:** Mathematical models do not always account for ethical considerations or strategic implications.
- **Example:** A model might recommend a cost-cutting measure that involves layoffs to improve profitability. While the model may show financial benefits, it does not consider the potential negative impact on employee well-being or company reputation.

7. Time Sensitivity

- **Limitation:** Mathematical models may become outdated if they are not regularly updated to reflect current conditions.
- **Example:** A financial model based on historical interest rates may not be accurate if there is a significant change in interest rates due to economic shifts or policy changes.

8. Limited Scope

- **Limitation:** Some mathematical models may only address specific aspects of business operations and may not provide a comprehensive view.
- **Example:** A model focusing solely on cost minimization might not account for factors like market demand, customer preferences, or competitive dynamics, leading to incomplete strategic recommendations.

Conclusion

Business Mathematics is an essential discipline that bridges the gap between theoretical mathematics and practical business applications. Business Mathematics provides essential tools for analyzing and solving business problems. By understanding and applying mathematical techniques, businesses can solve complex problems, make informed decisions, and achieve their objectives more effectively. Whether it's calculating costs, analyzing data, or optimizing operations, Business Mathematics provides the tools needed to succeed in a competitive environment. Understanding these mathematical principles is essential for effective business management, strategic planning, and operational efficiency.

Task 1

MASTERY EXERCISE 1.1

1. True and False

   i. Business mathematics includes the study of shapes and sizes.
   Answer: False (This is covered by geometry, which is part of mathematics but not specifically business mathematics.)

   ii. Linear programming is used to optimize resource allocation and minimize costs.

Answer: True

iii. Advanced mathematical techniques are always straightforward and do not require specialized knowledge.
Answer: False (Advanced techniques can be complex and require specialized knowledge.)

iv. Mathematical models in business should be updated regularly to remain relevant in dynamic environments.
Answer: True

2. Dragging (Match the Following)

i. Match each term with its corresponding description:

a) Financial Mathematics
b) Algebra in Business
c) Statistics and Data Analysis
d) Optimization and Operations Research
e) Calculus in Business
f) Decision Analysis

Descriptions:

a) Determining the optimal inventory levels to minimize costs
b) Calculating the average income of a group of people
c) Evaluating the marginal cost or revenue of producing an additional unit
d) Using equations to find the break-even point
e) Making decisions under uncertainty using decision trees
f) Calculating interest on a loan

Answers:

a) Financial Mathematics - f. Calculating interest on a loan
b) Algebra in Business - d. Using equations to find the break-even point
c) Statistics and Data Analysis - b. Calculating the average income of a group of people
d) Optimization and Operations Research - a. Determining the optimal inventory levels to minimize costs
e) Calculus in Business - c. Evaluating the marginal cost or revenue of producing an additional unit
f) Decision Analysis - e. Making decisions under uncertainty using decision trees

Activity 4

You are required to watch the video on Importance of Mathematics in Business. After watching, please create a visual representation of the key points from the video.

VIDEO 1: Importance of Mathematics in Business

Please click on the link below: https://www.youtube.com/watch?v=IC6Eb5HuIP8

Activity 5:

ONLINE DISCUSSION

Use chats, forum, question/answer, message my teacher to engage others.

Discuss why mastering basic concepts in Business Mathematics is crucial for understanding more advanced topics later on. How do these fundamental skills support your ability to grasp more complex mathematical techniques used in business?

Post your work in the discussion forum.

Activity 6

END OF MODULE ASSESSMENT 1

Multiple-Choice Questions

1. Which of the following best describes the primary goal of a business?

A) To produce goods without considering profit.

B) To create value for customers and generate profit for owners.

C) To offer free services to the community.

D) To focus solely on reducing costs.

Answer: B) To create value for customers and generate profit for owners.

---

2. If a small bakery wants to calculate the total cost of its ingredients for the month and the cost of flour is KES 200, sugar is KES 150, and other ingredients are KES 100, what is the total cost?

A) KES300

B) KES400

C) KES450

D) KES500

Answer: C) KES450

---

3. In the context of Business Mathematics, which mathematical concept is commonly used to determine the point at which total revenue equals total costs?
A) Financial mathematics
B) Algebra
C) Statistics
D) Optimization
Answer: B) Algebra

---

4. When analyzing market trends and customer preferences, which mathematical technique helps businesses make informed decisions based on collected data?
A) Algebra
B) Calculus
C) Statistics
D) Linear programming
Answer: C) Statistics

REFERENCE LIST AND FURTHER READING

1. Gary Bronson, Richard Bronson, & Maureen Kieff. (2021). *Mathematics for Business*. Mercury Learning and Information. https://research.ebsco.com/linkprocessor/plink?id=0b9336e0-e2fc-3560-9c10-c5c674163b70
2. BBP Learning Media. (2012). *CIMA Fundamentals of Business Mathematics Revision Kit* (Vol. 00003). BPP Learning Media. https://research.ebsco.com/linkprocessor/plink?id=df1c1006-5fc3-3cd2-8fdc-70aa8c0d1872
3. Mukras, M. S. (2014). *Elements of mathematical economics*. Kenya Literature Bureau.
4. Randy Cris Nicolas. (2020, October 22). *Importance of Mathematics in Business*. YouTube. https://www.youtube.com/watch?v=IC6Eb5HuIP8

TAKE HOME MESSAGE

Congratulations on completing Module One of your Introduction to Business Mathematics course! Reflect on how the basic mathematical skills you've learned in this module can influence your approach to solving business problems and making decisions in your future career.

WHAT'S NEXT?

- Module 1 Review Assignment
- Module 2: Basic Algebra and Arithmetic
- Welcome to this module as prepare to uncover the power of algebra and arithmetic, exploring how these concepts underpin critical business operations and decision-making!

MODULE 2: BASIC ALGEBRA AND ARITHMETIC

INSTRUCTIONAL HOURS: 4

MODULE OVERVIEW

In this module, you will explore the foundational concepts of algebra and arithmetic, which are essential tools in various fields, including business, economics, and the sciences. We will cover the properties of real numbers, ensuring a solid understanding of how they form the basis for arithmetic operations. You'll learn to manipulate algebraic expressions through various operations, gaining skills necessary for solving complex equations. The module will also guide you through linear equations and inequalities, focusing on their practical applications in real-world scenarios. Finally, you will apply these mathematical concepts in business contexts, such as break-even analysis, enabling you to make informed decisions based on quantitative data. By the end of this module, you will have developed the analytical skills required to solve problems and apply algebraic reasoning in both academic and professional settings.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Define real numbers, their classification and the fundamental operations that apply to them.
2. Demonstrate proficiency in performing operations with algebraic expression.
3. Formulate and solve mathematical problems derived from real business scenario.
4. Critically evaluate the role of algebra and arithmetic in problem-solving.

LEARNING ACTIVITIES/TASK LIST

1. Review PDF notes
2. Masterly exercise 1.1
3. Watch video
4. Complete participation assignment: Quiz/questions 1
5. Reading material 1: Notes
6. Review concept assignment : Quiz/questions 2
7. Complete end of module assessment

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning

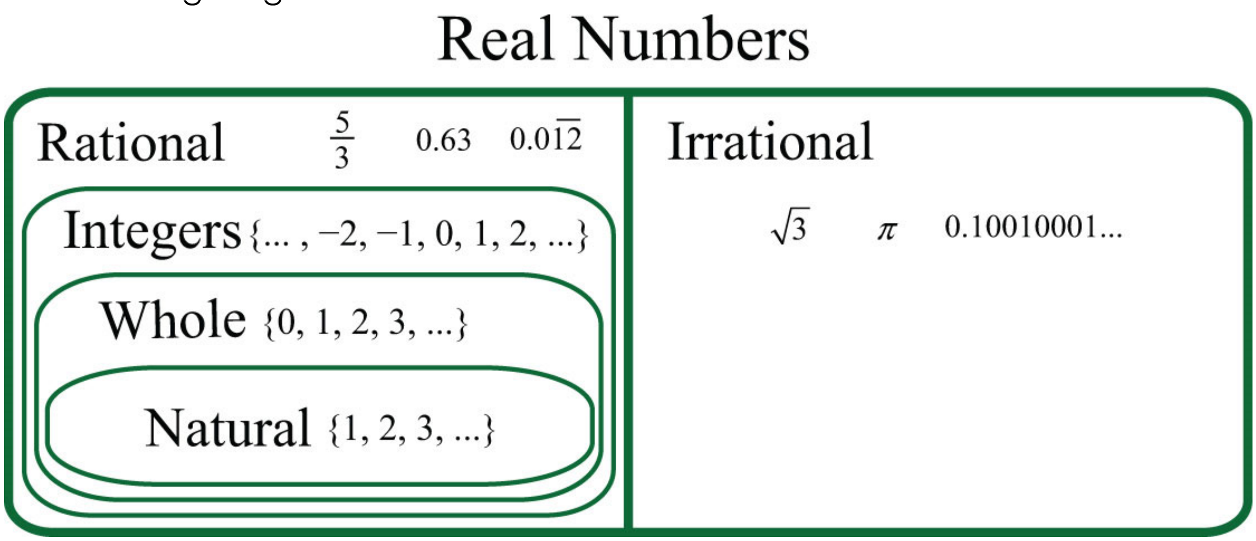
READING MATERIAL 1

(Notes, presentation, an article, a book chapter, case study, video, please add assessment…)

Activity 1
You are required to read the notes on real number system.
Upon completion; undertake the quiz or questions **(Task 1)**

REAL NUMBER SYSTEM
The concept of real number system evolved over time by expanding the idea of what we mean by the word "number." At first, "number" meant something you could count, like how many goats a farmer owns. These are called the natural numbers, or sometimes the counting numbers.

Real numbers include all the numbers on the number line, including both rational and irrational numbers. Real numbers can be categorized into several subsets:
Types of Numbers
The set of real numbers are made up of types of numbers as can be represented by the following diagram.



Definitions
Natural Numbers (N)
These are the numbers used for counting such as; 1, 2, 3, 4, 5 . . . and they can be prime or composite numbers.
A natural number is a prime number if it is greater than 1 and its only factors are 1 and itself. A natural number is a composite number if it is greater than 1 and it is not prime.

Example: 5, 7, 13, 29, 31 are prime numbers. 8, 24, 33 are composite numbers.

Whole Numbers (W)
At some point, the idea of "zero" came to be considered as a number. For instance if the farmer does not have any goat, then the number of goats that the farmer owns is zero.
We call the set of natural numbers plus the number zero the whole numbers i.e. 0, 1, 2, 3, 4, 5 . . .

Integers (Z)
The integers consist of the natural numbers together with the negatives and 0: ..., -4, -3, -2, -1, 0, 1, 2, 3, 4...
In other words, the integers are all the whole numbers and their additive inverses. As you may note, no fractions or decimals in this case.
An integer is even if it can be written in the form; $2n$, where n is an integer (if 2 is a factor).
An integer is odd if it can be written in the form; $2n - 1$, where n is an integer (if 2 is not a factor).
Example: 2, 0, 8, -24 are even integers and 1, 57, -13 are odd integers.

Rational numbers (Q)
The rational numbers are the numbers that can be written as the ratio of two integers. All rational numbers when written in their equivalent decimal form will have terminating or repeating decimals.
Therefore rational numbers are all numbers of the form: $\frac{a}{b}$ $but\ b \neq 0$. Rational numbers include what we usually call fractions.
Note that the word "rational" contains the word "ratio," which should remind you of fractions.
The bottom part of the fraction is called the denominator.
Think of it as the denomination which tells you what size fraction we are talking about: fourths, fifths, etc.
The top part of the fraction is called the numerator. It tells you how many fourths, fifths, or whatever.

Irrational Numbers ($\overline{Q}$)
The irrational numbers are any real numbers that cannot be represented as the ratio of two integers. Irrational numbers are usually imperfect roots such as Pi (π) and$\sqrt{2}$.
Irrational numbers when written in their equivalent decimal form have non-terminating and non-repeating decimals. The square root of a prime number is irrational.

Examples: $\sqrt{3}$, $\sqrt[3]{5}$, π, e

Real numbers (R)
When we put the irrational numbers together with the rational numbers, we finally have the complete set of real numbers.
Any number that represents an amount of something, such as a weight, a volume, or the distance between two points, will always be a real number.
A real number is positive if it is greater than 0, negative if it is less than 0.

Properties of Real Numbers

i) **Commutative Property**

For Addition: a + b = b + a
For Multiplication: ab = ba

ii) **Associative Property**

For Addition: (a + b) + c = a + (b + c)

For Multiplication: (ab)c = a(bc)

iii) Distributive Property

a(b + c) = ab + ac or (b + c)a = ab + ac

iv) Identity Property

a + 0 = 0 + a = a

v) Inverse Property

$$a + (-a) = 0 \text{ and } a \times \frac{1}{a} = 1 \text{ for } a \neq 0$$

Subtraction

This is the inverse operation for addition ("undoes" addition) and is the same as adding the negative of the number to be subtracted: a – b = a + (-b)

Properties of Negatives

1. (-1)a = -a
2. –(-a) = a
3. (-a)b = a(-b) = -(ab)
4. (-a)(-b) = ab
5. –(a + b) = -a – b
6. –(a – b) = b – a

Multiplicative Identity

a · 1 = 1 ·a = a

Division

This is the inverse operation for multiplication ("undoes" multiplication) and is the same as multiplying by the reciprocal: $a \div b = a.\frac{1}{b}$

Properties of Fractions

1. $\frac{a}{b}.\frac{c}{d} = \frac{ac}{bd}$
2. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$
3. $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$
4. $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$
5. $\frac{ac}{bc} = \frac{a}{b}$
6. If $\frac{a}{b} = \frac{c}{d}$,then $ad = bc$

Ordered Set

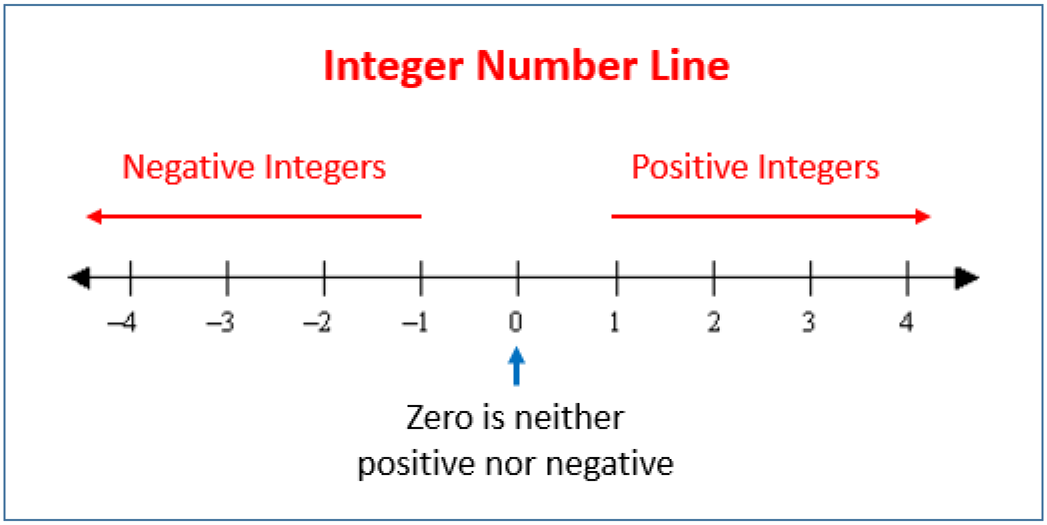
The real numbers have the property that they are ordered, which means that given any two different numbers we can always say that one is greater or less than the other.

A more formal way of saying this is: For any two real numbers a and b, one and only one of the following three statements is true:

1. a is less than b, (expressed as a < b)
2. a is equal to b, (expressed as a = b)
3. a is greater than b, (expressed as a > b)

The Number Line

The ordered nature of the real numbers allows us to arrange them along a line (imagine that the line is made up of an infinite number of points all packed so closely together that they form a solid line). The points are ordered so that points to the right are greater than points to the left:



From the figure above, every real number corresponds to a distance on the number line, starting at the center (zero). The negative numbers represent distances to the left of zero, and positive numbers are distances to the right.
Further, the arrows on the end indicate that it keeps going forever in both directions.

Absolute Value

The absolute value of a number is the distance from that number to the origin (zero) on the number line and denoted as $|a|$.
That distance is always given as a non-negative number.
Formal definition;

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a > 0 \end{cases}$$

Task 1

MASTERY EXERCISE 1.1

1. Which of the following numbers is an example of a natural number?

   A) -3

   B) 0

   C) 7

   D) 0.5

   Answer: C) 7

2. Which of the following sets includes all whole numbers and their negatives?

A) Natural Numbers

B) Whole Numbers

C) Integers

D) Rational Numbers

Answer: C) Integers

3. Which of the following is an irrational number?

A) 1/2

B) 0.333...

C) √2

D) 3/4

Answer: C) √2

4. Which property is illustrated by the equation a+b=b+a?

A) Associative Property

B) Commutative Property

C) Distributive Property

D) Inverse Property

Answer: B) Commutative Property

5. What is the absolute value of -7?

A) -7

B) 0

C) 7

D) -1

Answer: C) 7

1. What key concepts do classical economic theories, as introduced by Adam Smith and David Ricardo, emphasize regarding economic development? a) The role of government intervention
   b) The importance of human capital
   c) Capital accumulation and free markets
   d) Technological innovation

Activity 2

You are required to watch the video on **Basics of Algebraic Expressions: Simplification and Examples** which provides an introduction to algebraic expressions, covering fundamental concepts such as combining like terms, applying the distributive property, and simplifying expressions with step-by-step examples. After watching, attempt Quiz/questions labeled **task 2.**

VIDEO 1: Basics of Algebraic Expressions: Simplification and Examples

Please click on the link below:
https://youtu.be/OF2GtlinL_s?si=hAy35YpUy8WRy41q

Task 2

QUIZ/ QUESTIONS 1

1. What is an algebraic expression? Provide an example?
2. How do you apply the distributive property to the expression 4(2x+3)?
3. What are like terms, and how do they affect the simplification of an expression?
4. Simplify the expression 2a+3b+4a−b
5. In the expression 5(x+2)−3x what operation must be performed before combining like terms?

Answers

1. An algebraic expression is an expression that contains numbers, operations, and one or more variables.
2. Multiply 4 by each term inside the parentheses: 4×2x+4×3=8x+12
3. Like terms are terms that have the same variable raised to the same power. They are combined by adding or subtracting their coefficients.
4. Combine the like terms 2a and 4a, and 3b and −b to get 6a+2b.
5. First, distribute the 5 across x+2x+2 to get 5x+10, then subtract 3x to get the simplified expression 2x+10.

Activity 3

READING MATERIAL 1

You are required to Review the Notes on **Linear Equations and Inequalities**

Upon completion; undertake the quiz or questions (**Task 3)**

Linear Equations

Definition: A linear equation is an equation that can be written in the form

$$ax + b = c$$

where a, b, and c are constants and x is the variable.

Example

$2x + 5 = 11$

Solving Linear Equations

Isolate the variable: Use inverse operations (addition, subtraction, multiplication, and division) to get the variable alone on one side of the equation.

Check the solution: Substitute the solution back into the original equation to verify that it makes the equation true.

Example

Solve for x in the equation $3x - 7 = 14$.

Solution

Add 7 to both sides: $3x = 21$

Divide both sides by 3: $x = 7$

Graphing Linear Equations

A linear equation represents a straight line on a coordinate plane. To graph a linear equation, you need two points that satisfy the equation.

One common method is to find the x-intercept and the y-intercept.

x-intercept: The point where the line crosses the x-axis. To find the x-intercept, set y = 0 and solve for x.

y-intercept: The point where the line crosses the y-axis. To find the y-intercept, set x = 0 and solve for y.

Slope-Intercept Form

Another way to represent a linear equation is in slope-intercept form:

$y = mx + b$, where m is the slope and b is the y-intercept.

Slope: The slope measures the steepness and direction of the line. It is calculated as the rise over run between two points on the line.

Y-intercept: The point where the line crosses the y-axis.

Linear Inequalities

Definition: A linear inequality is a mathematical statement that compares two expressions using an inequality symbol ($<, >, \leq, \geq$)

Example: $2x + 5 < 11$

Solving Linear Inequalities

The process of solving linear inequalities is similar to solving linear equations, with one important exception: If you multiply or divide both sides by a negative number, you must reverse the inequality symbol.

Example:

Solve for x in the inequality $-3x + 7 \geq 14$

Subtract 7 from both sides: $-3x \geq 7$

Divide both sides by -3 (and reverse the inequality): $x \leq -7/3$

Graphing Linear Inequalities

To graph a linear inequality, first graph the corresponding linear equation. Then, shade the region that satisfies the inequality.

< or >: Shade the region below the line.

≤ or ≥: Shade the region above the line.

Applications of Linear Equations and Inequalities

Linear equations and inequalities have many real-world applications, including:

1. Business: Calculating profit, loss, and break-even points.
2. Science: Modeling relationships between variables, such as temperature and pressure.
3. Engineering: Designing structures and systems.
4. Economics: Analyzing supply and demand.

By understanding linear equations and inequalities, you can solve a variety of problems and make informed decisions. This will be covered extensively in the subsequent modules.

Task 3

QUIZ/ QUESTIONS 2

1. Which of the following is the correct solution for the linear equation $4x - 8 = 2x + 10$

   A) $x = -9$
   B) $x = 9$
   C) $x = -5$
   D) $x = 5$
   Answer: D) $x = 5$

2. If a line has a slope of $-\frac{3}{4}$ and passes through the point (2, 5), which of the following is the equation of the line in slope-intercept form?

   A) $y = -\frac{3}{4}x + \frac{19}{4}$

   B) $y = \frac{3}{4}x + \frac{13}{4}$

   C) $y = -\frac{3}{4}x + \frac{29}{4}$

   D) $y = -\frac{3}{4}x + -\frac{19}{4}$

   Answer: C) $y = -\frac{3}{4}x + \frac{29}{4}$

3. True or False: The solution to the inequality $-5x + 12 \leq 2x - 7$ includes negative values of x.

   Answer: True

4. True or False: When solving the inequality $-4x > 8$ by dividing both sides by −4, the inequality sign remains the same.

   Answer: False (The inequality sign should be reversed)

Activity 5

END OF MODULE ASSESSMENT 1.2

1. Using appropriate properties, simplify the following expression: 3(a+2)−4(a−5)

   Answer: −a+26

2. Explain the commutative and associative properties of real numbers, providing an example for each.

   Answer:

   Commutative Property: Changing the order of numbers does not affect the sum or product.

   Example: a+b=b+a or ab=ba

   Associative Property: Grouping of numbers does not affect the sum or product.

   Example: (a+b)+c=a+(b+c) or (ab)c=a(bc)

3. Solve the inequality 3x−5>2(x+1). Represent your solution on a number line.

   Answer: x>7

   On the number line, an open circle at 7 with a line extending to the right

4. For the inequality 4x−9≤2x+3, solve for xx and explain how inequalities differ from equations when graphing solutions.

   Answer: x≤6

Difference: Inequalities use open or closed circles depending on whether the inequality includes equality (≤, ≥) and arrows to show ranges.

5. A business owner has 150 units of a product. They plan to sell the product for \$20 per unit and incur a fixed cost of \$500. Formulate and solve a linear equation to determine the number of units they need to sell to break even.
   Equation: $20x=500$
   Divide by 20
   Answer $x=25$
   Solution: They need to sell 25 units to break even.
6. Simplify the algebraic expression $6x^2+3x-2(2x^2-5x+4)$. Clearly show all steps involved in combining like terms.
   Answer: $2x^2+13x-8$

REFERENCE LIST AND FURTHER READING

1. Gary Bronson, Richard Bronson, & Maureen Kieff. (2021). *Mathematics for Business*. Mercury Learning and Information.
https://research.ebsco.com/linkprocessor/plink?id=0b9336e0-e2fc-3560-9c10-c5c674163b70
2. BBP Learning Media. (2012). *CIMA Fundamentals of Business Mathematics Revision Kit* (Vol. 00003). BPP Learning Media.
https://research.ebsco.com/linkprocessor/plink?id=df1c1006-5fc3-3cd2-8fdc-70aa8c0d1872
3. Mukras, M. S. (1986). *Elements of mathematical economics*. Kenya Literature Bureau.
4. Algebraic Expressions (Basics. (2014, September 24). *Algebraic Expressions (Basics)*. YouTube. https://youtu.be/OF2GtlinL_s?si=hAy35YpUy8WRy41q

TAKE HOME MESSAGE

Congratulations on completing the Basic Algebra and Arithmetic module! As you move forward, challenge yourself to apply these fundamental concepts to real-world business scenarios. Think about how understanding real numbers, algebraic expressions, and linear equations can enhance your problem-solving skills.

WHAT'S NEXT?

- Module 2 Review Assignment
- Module 3: Functions and Graphs
- Get ready to embark on an exciting journey into the world of Functions and Graphs! In this module, you'll unlock the secrets behind how different variables interact and influence each other. Prepare to be inspired and amazed as you uncover the power of functions and graphs to reveal hidden patterns and drive impactful decisions. Let's make this a thrilling adventure into the heart of mathematics!

MODULE 3: FUNCTIONS AND GRAPHS

INSTRUCTIONAL HOURS: 3

MODULE OVERVIEW

In this module, you will delve into the essential concepts of functions and graphs, which are critical for understanding relationships between variables in various fields, especially in business and economics. We will begin by defining and exploring different types of functions, including linear, quadratic, exponential, and logarithmic functions. You will learn how to graph these functions, gaining the ability to visually interpret and analyze data. The module will also cover the practical applications of functions in business, such as modeling cost, revenue, and profit functions, allowing you to apply mathematical concepts to real-world scenarios. By the end of this module, you will develop the skills needed to effectively utilize functions and graphs in solving problems and making data-driven decisions in business environments.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Identify and define different types of function.
2. Graph and interpret linear, quadratic, exponential, and logarithmic functions.
3. Formulate and apply functions to model business scenarios using mathematical techniques to solve practical problems.
4. Evaluate the importance of functions and graphs in making informed, data-driven decisions in business and economics.

LEARNING ACTIVITIES/TASK LIST

1. Review power presentation PDF
2. Complete participation assignment: Quiz/questions 1
3. Reading material 1
4. Watch video
5. Quiz/questions 2
6. Masterly exercise 1.1
7. End of module assessment 1.1

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning
READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment…)

Activity 1

Review the power point Notes on graphs and functions; undertake the quiz or questions labeled **(Task 1)**

GRAPHS AND FUNCTIONS

Suppose that the average weekly household expenditure on food (C) depends on average net household weekly income (*Y*) according to the relationship;

C = 12 + 0.3*Y*

This implies that for any given value of Y, one can evaluate what C will be.

For example if Y = 90 then;

C = 12 + 27 = 39

Whatever value of Y is chosen there will be one unique corresponding value of C. This is an example of a function.

A relationship between the values of two or more variables can be defined as a function when a unique value of one of the variables is determined by the value of the other variable or variables.

If the precise mathematical form of the relationship is not actually known then a function may be written in what is called a general form.

For example, a general form demand function is

$$Qd = f(P)$$

This particular general form just tells us that quantity demanded of a good; Qd depends on its price (P). The "f" is not an algebraic symbol in the usual sense and so f(P) means "is a function of P" and not "f multiplied by P".

In this case P is what is known as the **"independent variable"** because its value is given and is not dependent on the value of Qd, i.e. it is exogenously determined. On the other hand Qd is the **"dependent variable"** because its value depends on the value of P.

Functions may have more than one independent variable. For example, the general form production function;

$Q = f(K, L)$ tells us that output (Q) depends on the values of the two independent variables capital (K) and labour (L).

The specific form of a function tells us exactly how the value of the dependent variable is determined from the values of the independent variable or variables.

A specific form for a demand function might be;

$Qd = 120 - 2P$

For any given value of P the specific function allows us to calculate the value of Qd.

For example;

when P = 10 then Qd = 120 − 2(10) = 120 − 20 = 100

when P = 45 then Qd = 120 − 2(45) = 120 − 90 = 30

In business applications of functions it may make sense to restrict the 'domain' of the function, i.e. the range of possible values of the variables.

For example, variables that represent price or output may be restricted to positive values. Strictly speaking the domain limits the values of the independent variables and the range governs the possible values of the dependent variable.

When defining the specific form of a function it is important to make sure that only one unique value of the dependent variable is determined from each given value of the independent variable(s).

Consider the equation

$$y = 80 + x^{0.5}$$

This does *not* define a function because any given value of *x* corresponds to two possible values for *y*.

For example, if x = 25, then $25^{0.5} = 5$ or −5 and so *y* = 75 or 85.

However, if we define;

$y = 80 + x^{0.5}$; for $x^{0.5} \geq 0$ then this does constitute a function.

When domains are not specified then one should assume a sensible range for functions representing business and economic variables.

Inverse functions

An inverse function reverses the relationship in a function.

If we confine the analysis to functions with only one independent variable, x, this means that if y is a function of x, i.e.

$$y = f(x)$$

then in the inverse function x will be a function of y, i.e.

$$x = g(y)$$

(The letter g is used to show that we are talking about a different function.)

Example

If the original function is

$$y = 4 + 5x$$

then;

$$y - 4 = 5x$$

$$0.2y - 0.8 = x$$

and so the inverse function is;

$$x = 0.2y - 0.8$$

Exercise

Determine the inverse of the following function

$$y = \frac{2x+3}{x-4}$$

Answer: $x = \frac{4x+3}{x-2}$

Not all functions have an inverse function. The mathematical condition necessary for a function to have a corresponding inverse function is that the original function must be "monotonic".

This means that, as the value of the independent variable x is increased, the value of the dependent variable y must either always increase or always decrease. It cannot first increase and then decrease, or vice versa. This will ensure that, as well as there being one unique value of y for any given value of x, there will also be one unique value of x for any given value of y.

Example

Consider the function $y = 9x - x^2$ restricted to the domain 0 ≤ x ≤ 9.

Each value of x will determine a unique value of y. However, some values of y will correspond to two values of x, e.g.

when x = 3 then y = 27 − 9 = 18

when x = 6 then y = 54 − 36 = 18

This is because the function $y = 9x - x^2$ is not monotonic. This can be established by calculating y for a few selected values of x:

X	1	2	3	4	5	6	7
Y	8	14	18	20	20	18	14

These figures show that y first increases and then decreases in value as x is increased and so there is no inverse for this non-monotonic function.

Although mathematically it may be possible to derive an inverse function, it may not always make sense to derive the inverse of an economic function, or many other business functions that are based on empirical data.

For example, if we take the geometric function that the area A of a square is related to the length L of its sides by the function $A = L^2$, then we can also write the inverse function that relates the length of a square's side to its area: $L = A^{0.5}$ (assuming that L can only take non-negative values). Once one value is known then the other is determined by it.

However, suppose that someone investigating expenditure on holidays abroad (H) finds that the level of average annual household income (M) is the main influence and the relationship can be explained by the function

H = 0.01M + 100 for M ≥ £10, 000

This mathematical equation could be rearranged to give

M = 100H − 10, 000

but to say that H determines M obviously does not make sense. The amount of holidays taken abroad does not determine the level of average household income.

From the forgoing discussions, we can then summarize that a function is well defined by an equation where an equation is a mathematical relationship that shows equality between the left and the right hand side e.g.

$$y = a + bx$$

Where y and x are variables, a and b are constants or parameters.

The collection of all the values of independent variables for which a function is defined is referred to as domain of the function.

Values that $y = 4 + 4x$; $x = 1,2$ then they are said to be domain of a function.

The range of the function is a collection of a corresponding values of the independent variables

Example

$$y = 4 + 4x$$

Where x = 1, 2

When x = 1; y = 4 + 4(1) =8

When x = 2; y = 4+4(2) = 12

Then 8 and 12 are the range

A function is said to be continuous if there are no gaps in the function over a range of functions of possible values.

A continuous variable is one that can assume a series of values between any two given values. E.g. height, weight and distance

A function is not continuous if it is measured in jumps between any two variables.

A function of the nature;

$$y = a + bx$$

Where y and x are variables a and b are constraints is called explicit function or equation. In an explicit function, you can be able to pick dependent (y) and the independent variable (x)

It should be noted that there are certain functions where one cannot be able to differentiate between the dependent and the independent variable. E.g.

$$2x^2 + 8x + y = 4$$

They are called implicit functions

We can be able to form explicit function from implicit functions just by taking one variable at a time and making it the subject if the function.
The number of implicit functions that can be formed from implicit function will be equal to the number of variables in implicit function.

Types of functions

1. Linear Function

This are functions of first degree. A function where the highest power to which any variable is raised to is one e.g.

$$y = a + bx$$

2. Quadratic Functions

This is a function of second degree. The highest power to which a any variable is raised to is 2 e.g.

$$y = ax^2 + bx + c$$

3. Cubic functions

These are functions of third degree. The highest power to any variable is raised to is three.

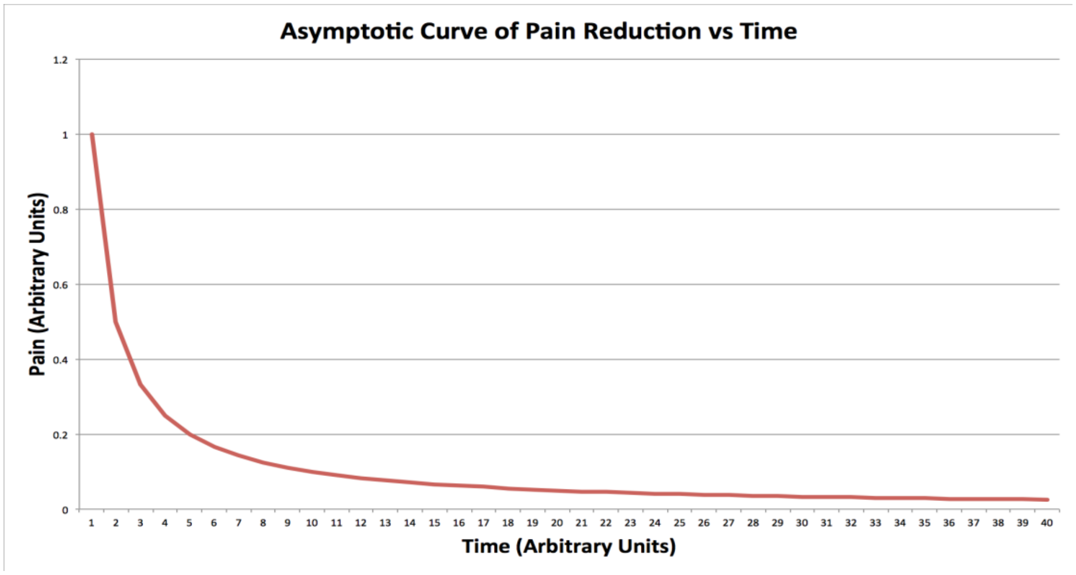
$$y = ax^3 + bx^2 + cx + k$$

4. Rectangular Hyperbolic functions

They are the functions of the nature of;

$$y = \frac{k}{x}$$

Where y and x are variables and k is a constant.
These functions when plotted in a Cartesian plane, they will give a curve that is asymptotic to a given axis or line.
Asymptotic is a line that the curve approaches but it doesn't touch the axis



5. Exponential functions

These are functions of the nature $y = a^x$ where y and x are variables and a is a constant.

When plotted in a Cartesian plane, they will be asymptotic to a given axis

They can also be presented as;

$$y = e^x$$

Where e is 2.714 …

6. Logarithmic functions

These are functions which involve log

$$y = \log_a X$$

Where x and y are variables and a is constant. This type of function will also give a curve that is asymptotic to a given axis.

LINEAR FUNCTIONS

As already stated, this are functions of first degree.

$$y = a + bx$$

Graphical presentation

Ways of plotting linear functions includes;

a) Tabular method

b) Intercept method

Tabular method

Make a table using the equation i.e.

X	1	0	-1
Y	-2	-4	-6

Choose a suitable scale and plot the line of the equation

Intercept method

Identify the point where line cuts the x and y axis.

The point where line cuts the y axis; x = 0

The point where line cuts the x axis; y = 0

Therefore; at y-intercept x = 0 and at x-intercept y = 0

Slope of linear functions

Given;

$$y = a + bx$$

a = y-intercept and b = gradient

Slope of line PQ is change in vertical displacement to change in horizontal displacement. i.e.

$$Slope = \frac{change\ in\ vertical\ displacement}{change\ in\ horizontal\ displacement} = \frac{y_2 - y_1}{x_2 - x_1}$$

SIMULTANEOUS LINEAR EQUATIONS

The basic idea involved in all the different methods of algebraically solving simultaneous linear equation systems is to manipulate the equations until there is a single linear equation with one unknown. This can then be solved and the value of the variable that has been found can then be substituted back into the other equations to solve for the other unknown values.

It is important to realize that not all sets of simultaneous linear equations have solutions.

The general rule is that the number of unknowns must be equal to the number of equations for there to be a unique solution.

However, even if this condition is met, one may still come across systems that cannot be solved, e.g. functions which are geometrically parallel and therefore never intersect.

We shall first consider four different methods of solving a 2 × 2 set of simultaneous linear equations, i.e. one in which there are two unknowns and two equations, and then look at how some of these methods can be employed to solve simultaneous linear equation systems with more than two unknowns.

Activity 2

You are required to watch the video on **Solving Systems of Linear Equations by Graphing.**

The video provides step-by-step examples of how to solve systems of linear equations by graphing, and it also explains the different interpretations of the solutions (one solution, no solution, or infinitely many solutions).

After watching, make short notes for your revision and attempt Quiz/questions labeled **task 2 and share your answers with your peers.**

VIDEO 1: Solving Systems of Linear Equations by Graphing

Please click on the link below:

https://youtu.be/SoVUECpWkKc?si=CoqcaT1qdHb85W_j

Task 2

QUIZ/ QUESTIONS 2

1. Solve for p and q in the set of simultaneous equations given below;

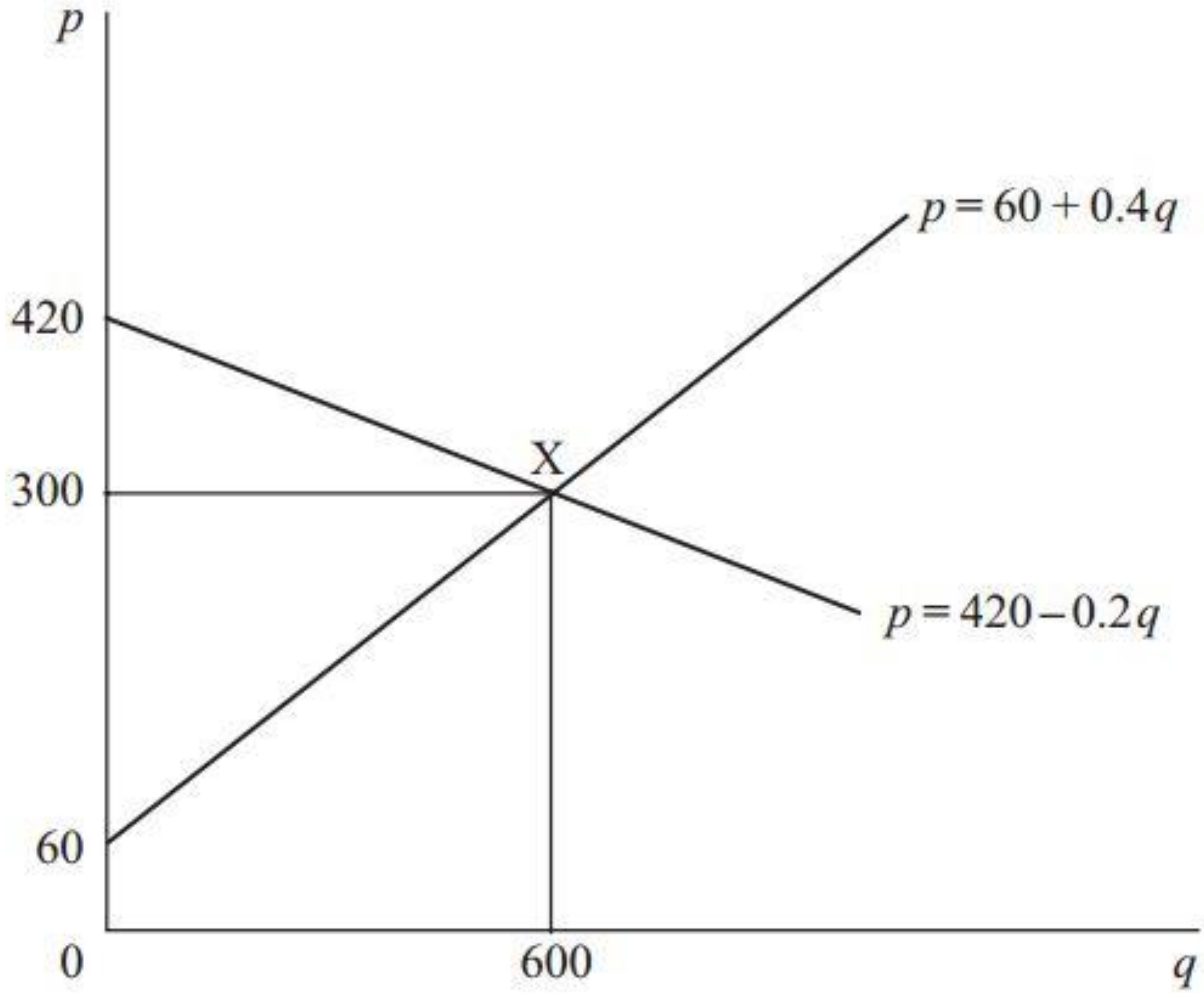
$p = 420 - 0.2q$

$p = 60 + 0.4q$

Solution

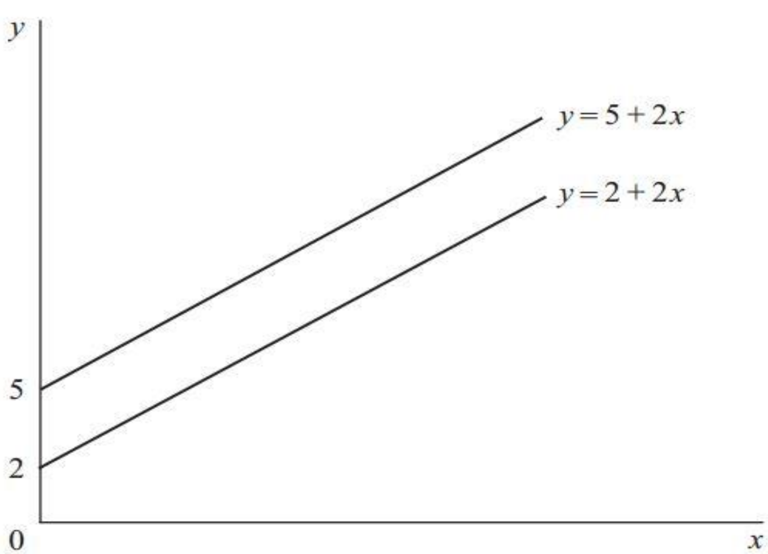
These two functional relationships are plotted as below. Both hold at the intersection point X. At this point the solution values

$p = 300$ and $q = 600$ can be read off the graph.



2. Attempt to use graphical analysis to solve for y and x if
$y = 2 + 2x$ and $y = 5 + 2x$

Solution



These two functions are plotted in the above figure. They are obviously parallel lines which never intersect. This problem therefore does not have a solution.

Activity 3
You are required to read the PowerPoint notes on **Elimination and Substitution methods for solving simultaneous linear equation systems**.
Upon completion; please undertake **Activity 4 on**

Elimination Method
A variable is eliminated by turn by making the absolute value of its coefficient to be the same in equation of the system and then adding or subtracting the equations.
Making the absolute values of the coefficients equal necessitates the multiplication of each equation by an appropriate numerical factor.
Example 1
Consider the system of two equations;
Attempt to use elimination method to solve for y and x if
$y = 2 + 2x$
$y = 5 + 2x$
Eliminating y from the system and equating the other two sides of the equations, we get
$2 + 2x = 5 + 2x$
Subtracting 2x from both sides gives 2 = 5. This is clearly impossible, and hence no solution can be found.
Example 2
Consider the system of two equations;
$2x – 3y = 8$…………………………………………………………………i
$3x + 4y = -8$………………………………………………………………ii

Solution
Step 1
Multiply (i) by 3 to get equation (iii)
Multiply (ii) by 2 to get equation (iv)
Subtract (iv) from (iii) to get equation (v)
Solve (v) foy y
Step 2
Multiply (i) by 4 to get (iii)
Multiply (ii) by 3 to get (iv)
Add (iii) to (iv) to get the value of x

Substitution Method
The substitution method involves rearranging one equation so that one of the unknown variables appears by itself on one side. The other side of the equation can then be substituted into the second equation to eliminate the other unknown.

Example

Solve the linear simultaneous equation system

$20x + 6y = 500$

$10x - 2y = 200$

Solution

Equation (2) can be rearranged to give

$10x - 200 = 2y$

$5x - 100 = y$ (3)

If we substitute the left-hand side of equation (3) for y in equation (1) we get

$20x + 6y = 500$

$20x + 6(5x - 100) = 500$

$20x + 30x - 600 = 500$

$50x = 1,100$

$x = 22$

To find the value of y we now substitute this value of x into (1) or (2). Thus, in (1)

$20x + 6y = 500$

$20(22) + 6y = 500$

$440 + 6y = 500$

$6y = 60$

$y = 10$

Activity 4

READING MATERIAL 1.1

You are required to Read Chapter 3, pages 101-114 of Essential Mathematics for Economics and Business by Bradley (2013). This chapter covers simultaneous equations, equilibrium, and breakeven analysis.

After reading, complete Progress Exercise 3.1in the book chapter and share your answers with your peers for discussion and feedback.

Activity 5

You are required to watch the video on **Cost, Revenue, Profit Equations and Break Even Point**

The video provides step-by-step illustration on how to write the cost function, revenue function, and profit function for a business. It also covers how to find the break-even point.

After watching, make short notes for your revision and attempt Quiz/questions labeled **task 3**

You Tube **VIDEO 2: Solving Systems of Linear Equations by Graphing**

Please click on the link below:

https://youtu.be/_wHuj937LS4?si=Ggq6c1_B2Z5FMOO6

Task 3

QUIZ/ QUESTIONS 2

A company produces handmade scarves. The fixed costs associated with production are $500 per month, and the variable cost per scarf is $10. The scarves are sold for $25 each.

(a) Write the cost function, revenue function, and profit function for this company.

(b) Find the break-even point for the company.

Solution

(a) Write the cost function, revenue function, and profit function for this company.

Cost function: C(x) = 500 + 10x, where x is the number of scarves produced.

Revenue function: R(x) = 25x, where x is the number of scarves sold.

Profit function: P(x) = R(x) - C(x) = 25x - (500 + 10x) = 15x - 500.

(b) Find the break-even point for the company.

To find the break-even point, we set the profit function equal to zero and solve for x:

15x - 500 = 0
15x = 500
x = 500/15
x ≈ 33.33

Activity 6

You are required to read the notes on Non-Linear functions.
Upon completion; you are to undertake (**Task 4**).

NON-LINEAR FUNCTIONS

QUADRATIC FUNCTIONS

Learning objectives

After completing this chapter students should be able to:

- Use factorization to solve quadratic equations with one unknown variable.
- Use the quadratic equation solution formula.
- Identify quadratic equations that cannot be solved.
- Set up and solve economic problems that involve quadratic functions.

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$ where a, *b*, and c are real numbers and $a \neq 0$.

A quadratic equation that includes terms in both x and x^2 cannot be rearranged to get a single term in x, so we cannot use the method used to solve linear equations.

SOLVING QUADRATIC EQUATIONS BY FACTORIZATION

This involves the process how some expressions can be broken down into terms which when multiplied together give the original expression. For example,

$a^2 - 2ab + b^2 = (a - b)\ (a - b)$

Example:

Solve the following quadratic equations by *factoring*.

a. $x^2 - 5x + 6 = 0$

b. $t^2 = 16$

c. $5z^2 = 2 - 9z$

SOLUTIONS:

a. $x^2 - 5x + 6 = 0$

$\Rightarrow (x - 3)(x - 2) = 0$

$\Rightarrow \quad x - 3 = 0$ or $x - 2 = 0$

$\Rightarrow \quad x = 3$ or $x = 2$

Thus, the solution set is $\{3, 2\}$

b. $t^2 = 16$

$\Rightarrow \quad t^2 - 16 = 0$

$\Rightarrow \quad (t + 4)(t - 4) = 0$

$\Rightarrow \quad t + 4 = 0$ or $t - 4 = 0$

$\Rightarrow \quad t = -4$ or $t = 4$

Thus, the solution set is $\{-4, 4\}$

c. $5z^2 = 2 - 9z$

$\Rightarrow \quad 5z^2 + 9z - 2 = 0$

$\Rightarrow (5z - 1)(z + 2) = 0$

$\Rightarrow \quad 5z - 1 = 0$ or $z + 2 = 0$

$\Rightarrow \quad 5z = 1$ or $z = -2$

$\Rightarrow \quad z = \frac{1}{5}$ or $z = -2$

Thus, the solution set is $\left\{\frac{1}{5}, -2\right\}$

SOLVING QUADRATIC EQUATIONS USING THE SQUARE ROOT METHOD

EXAMPLE:

Solve the following quadratic equations using *the square root method*.

a. $x^2 = 16$

b. $p^2 = -25$

c. $(y - 7)^2 = 9$

d. $(z + 3)^2 + 7 = 0$

SOLUTIONS:

a. $x^2 = 16$

$\Rightarrow \sqrt{x^2} = \pm\sqrt{16}$

$\Rightarrow x = \pm 4$

Thus, the solution set is $\{-4, 4\}$.

b. $p^2 = -25$

$\Rightarrow \sqrt{p^2} = \pm\sqrt{-25}$

$\Rightarrow p = \pm 5i$

Thus, the solution set is $\{5i, -5i\}$.

c. $(y-7)^2 = 9$

$\Rightarrow \sqrt{(y-7)^2} = \pm\sqrt{9}$

$\Rightarrow y - 7 = \pm 3$

$\Rightarrow y = 7 \pm 3$

$\Rightarrow y = 10$ or $y = 4$

Thus, the solution set is $\{10, 4\}$.

d. $(z+3)^2 + 7 = 0$

$\Rightarrow (z+3)^2 = -7$

$\Rightarrow \sqrt{(z+3)^2} = \pm\sqrt{-7}$

$\Rightarrow z + 3 = \pm i\sqrt{7}$

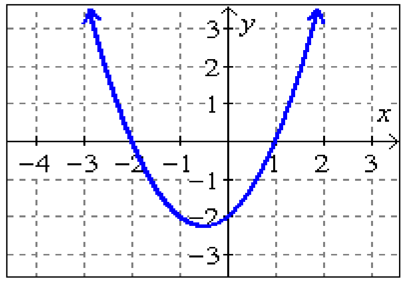
$\Rightarrow z = -3 \pm i\sqrt{7}$

Thus, the solution set is $\{-3 + i\sqrt{7}, -3 - i\sqrt{7}\}$.

SOLVING QUADRATIC EQUATIONS USING GRAPHS

Example:

Use the graph below to solve the quadratic equation; $x^2 + x - 2 = 0$.



The graph of $y = x^2 + x - 2$.

SOLUTION:

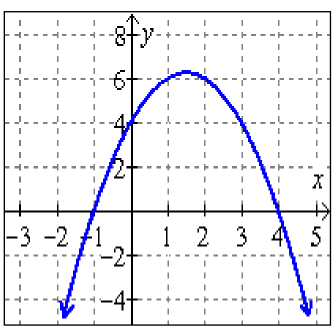
Since we are given the graph of $y = x^2 + x - 2$ and we are asked to solve $x^2 + x - 2 = 0$, we need to look for any places on the graph where $y = 0$, i.e., we need to find the x-intercepts of the graph.
Since the x-intercepts are $(-2, 0)$ and $(1, 0)$, we can conclude that the solutions are $x = -2$ or $x = 1$, so the solution set is $\{-2, 1\}$.

EXAMPLE:

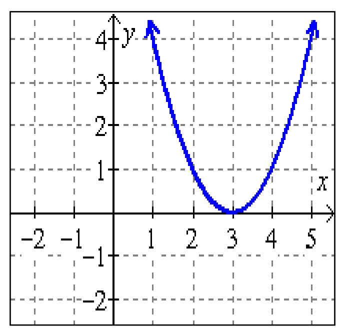
Use the graphs below to solve the given quadratic equations.

a. $0 = -x^2 + 3x + 4$

b. $x^2 - 6x + 9 = 0$



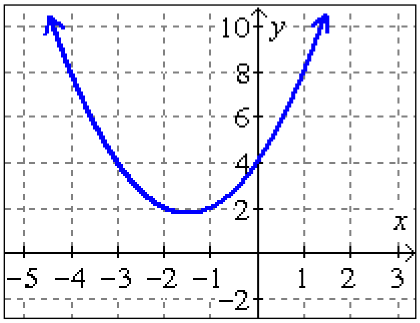
The graph of $y = -x^2 + 3x + 4$.



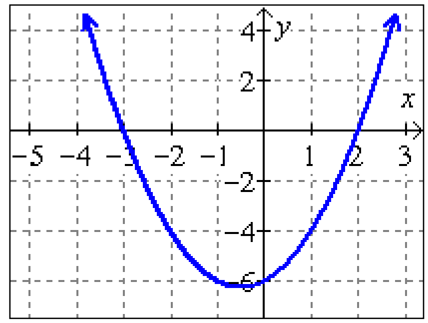
The graph of $y = x^2 - 6x + 9$.

c. $x^2 + 3x + 4 = 0$

d. $x^2 = 6 - x$



The graph of $y = x^2 + 3x + 4$.



The graph of $y = x^2 + x - 6$.

SOLUTIONS:

a. Since we are given the graph of $y = -x^2 + 3x + 4$ and we are asked to solve $0 = -x^2 + 3x + 4$, we need to look for any places where the graph of $y = -x^2 + 3x + 4$ intersects the line $y = 0$, i.e., we need to find the x-intercepts of the graph.

Since the x-intercepts are $(-1, 0)$ and $(4, 0)$, we can conclude that the solutions are $x = -1$ or $x = 4$, so the solution set is $\{-1, 4\}$.

b. Since the graph of $y = x^2 - 6x + 9$ intersects the line $y = 0$ only when $x = 3$, we can conclude that the solution set is $\{3\}$.

c. Since the graph of $y = x^2 + 3x + 4$ never intersects the line $y = 0$ (i.e., the x-axis), we see that "$x^2 + 3x + 4$" never equals 0, so we can conclude that the equation $x^2 + 3x + 4 = 0$ does not have any real solutions.

(Note that we can use the quadratic formula or completing-the-square to find the complex numbers that solve the equation.)

d. Here, we are given the graph of $y = x^2 + x - 6$ so we need to first obtain the expression "$x^2 + x - 6$" in the given equation $x^2 = 6 - x$ which we can do by solving this equation for 0:

$$x^2 = 6 - x$$

$$\Rightarrow \; x^2 + x - 6 = 0 \qquad \text{(subtract } 6 - x \text{ from both sides)}$$

We can solve this equation by looking for the x-values where the graph of $y = x^2 + x - 6$ intersects the line $y = 0$ (i.e., the x-axis). Thus, the solution set is $\{-3, 2\}$

SOLVING QUADRATIC EQUATIONS BY COMPLETING-THE-SQUARE

Using the square root method to solve a quadratic equation only works if we can write the quadratic equation so that one side is the square of a binomial. To write the square of a binomial, we start with a perfect square trinomial, i.e., a trinomial that can be factored into two identical binomial factors).

Perfect Square Trinomials	$\Rightarrow$	*Factored Form*
$x^2 + 8x + 16$	$\Rightarrow$	$(x+4)^2$
$x^2 - 6x + 9$	$\Rightarrow$	$(x-3)^2$

Notice that for each perfect square trinomial, the constant term of the trinomial is the square of half the linear term (in this case, the coefficient of x). For example,

$$x^2 + 8x + 16 \qquad\qquad x^2 - 6x + 9$$

$$\tfrac{1}{2}\cdot(8) = 4 \text{ and } 4^2 = 16 \qquad\qquad \tfrac{1}{2}(-6) = -3 \text{ and } (-3)^2 = 9$$

The process of writing a quadratic equation so that one side is a perfect square trinomial is called **completing the square**.

Example:

Solve $x^2 + 2x - 3 = 0$ for x by completing the square.

SOLUTION:

First, we need to find the square of half the coefficient of the linear term (in this case, the coefficient of x):

$$\tfrac{1}{2}\cdot(2) = 1 \text{ and } 1^2 = 1$$

Now we add and subtract 1 on the left-hand side of the equation. Notice that we are just adding zero in a disguised form, so we won't change the solution to the quadratic equation.

$$x^2 + 2x + 1 - 1 - 3 = 0 \quad \text{(add and subtract 1 on the left-hand side)}$$

$$\Rightarrow (x^2 + 2x + 1) - 1 - 3 = 0$$

$$\Rightarrow (x+1)(x+1) - 4 = 0$$

$$\Rightarrow (x+1)^2 - 4 = 0$$

$$\Rightarrow (x+1)^2 = 4$$

$$\Rightarrow \sqrt{(x+1)^2} = \pm\sqrt{4}$$

$$\Rightarrow x + 1 = \pm 2$$

$$\Rightarrow x = -1 \pm 2$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = -3$$

Thus, the solution set is $\{1, 3\}$.

In summary; To Solve a Quadratic Equation by Completing-the-Square:

1. Write the equation in standard form ($ax^2 + bx + c = 0$).
2. Divide both sides of the equation by the coefficient of the quadratic term, *a*, in order to make that coefficient 1.

3. Complete the square:

   a) Multiply the coefficient of the linear term by one-half, then square the result.
   b) Add and subtract the value obtained in step a
4. Write the square of a binomial plus a constant, then move the constant to the other side of the equation.
5. Use the square root method to find the solution.

Solve for the value of x using completing the square method;

$$ax^2 + bx + c = 0$$

Steps

Make the coefficient of x^2 unit i.e 1

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Add the square of half of the coefficient of x to both sides

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

Get the square root of both sides;

$$x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$$

Solve for x i.e. transpose $\frac{b}{2a}$ to the RHS

$$x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Solution by Quadratic formula

- Identify the constants a, b and c
- Put this values in the quadratic formulae;

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

- Solve for x

Task 3

MASTERY EXERCISE 1.1

1. Solve the quadratic equation $x^2-5x+6=0$ by factorization
   Answer: x=2 or x=3
2. Use the quadratic formula to solve $2x^2-4x-3=0$
   Answer: $x=1+\frac{\sqrt{10}}{2}$ or $x=1-\frac{\sqrt{10}}{2}$
3. Determine whether the quadratic equation $x^2+4x+5=0$ has real solutions.
   Answer: Since the discriminant is negative (Δ=-4<0), the equation has no real solutions. Instead, it has complex solutions.

4. Solve the quadratic equation $x^2-16=0$ using the square root method
   Answer: x=4 or x=−4
5. Solve the quadratic equation $x^2+6x=7$ by completing the square
   Answer: x=1 or x=-7

Task 4

END OF MODULE ASSESSMENT 1.2

1. Find the inverse of the function y=5x+2y. What is the value of xxx when y=12?

- A) 2

- B) 1.5
- C) 2.5
- D) 3

Answer: D) 3

2. Given the quadratic function f(x)=−3x²+6x+4, what is the maximum value of f(x)?
   A) 7
   B) 8
   C) 9
   D) 10
   Answer: C) 9
3. Solve the quadratic equation $x^2 - 5x + 6 = 0$ by factorization

   Answer: x=2 or x=3
4. Solve the following quadratic equations by *factoring*.

   $x^2 - 5x + 6 = 0$

   Answer=2
5. Solve for p and q in the set of simultaneous equations given below graphically;

p = 420 − 0.2q
p = 60 + 0.4q
Answer: p=300 and q=600

REFERENCE LIST AND FURTHER READING

1. BBP Learning Media. (2012). *CIMA Fundamentals of Business Mathematics Revision Kit* (Vol. 00003). BPP Learning Media. https://research.ebsco.com/linkprocessor/plink?id=df1c1006-5fc3-3cd2-8fdc-70aa8c0d1872
2. Bradley, T. (2013). *Essential mathematics for economics and business*. John Wiley & Sons. Pg 101-114.
3. Clendenen, G., & Salzman, S. A. (2018). *Business mathematics*. Pearson.
4. Gary Bronson, Richard Bronson, & Maureen Kieff. (2021). *Mathematics for Business*. Mercury Learning and Information. https://research.ebsco.com/linkprocessor/plink?id=0b9336e0-e2fc-3560-9c10-c5c674163b70
5. GoTutor Math. (2023, October 7). *Solving Systems of Linear Equations By Graphing | Algebra*. YouTube. https://www.youtube.com/watch?v=SoVUECpWkKc
6. Mario's Math Tutoring. (2023, November 1). *Cost, Revenue, Profit Equations and Break Even Point*. YouTube. https://www.youtube.com/watch?v=_wHuj937LS4
7. M. S. Mukras : (2004), *Elements of Mathematical Economics*, KLB, Nairobi
8. Ummer, E. K. (2012). *Basic mathematics for economics, business, and finance*. Routledge.

TAKE HOME MESSAGE

As you reflect on the power of graphs and functions in solving complex problems, think about how these mathematical tools can shape strategic decision-making in the real world.

How might you apply this knowledge to forecast business trends, optimize resource allocation, or even predict future outcomes in a rapidly changing economy? As you move forward into the next module, challenge yourself to connect these concepts to practical situations you'll encounter in your career.

WHAT'S NEXT?

- Module 3 Review Assignment
- Module 4: You will be learning about Matrix Algebra

Congratulations on completing the module on graphs and functions and discovering their valuable applications in business! Up next is Matrix Algebra, a powerful tool that will open doors to solving complex systems of equations, analyzing business networks, and optimizing decision-making processes. Prepare to explore how matrices can model real-world business scenarios, from managing inventories to analyzing market trends, and enhance your problem-solving toolkit.

Get ready to dive into a whole new dimension of business mathematics that will empower you to tackle larger challenges with confidence!

Happy learning, and let's make this next step even more rewarding!

MODULE 4: MATRIX ALGEBRA

INSTRUCTIONAL HOURS: 4

MODULE OVERVIEW

In this module, you will explore the fundamental concepts of matrix algebra, a powerful mathematical tool used in various fields, including business and economics. We will begin with an introduction to matrices, covering their definitions, types, and basic properties. You will learn how to perform essential matrix operations such as addition, subtraction, multiplication, and finding inverses. The module will also focus on practical applications of matrices in business, such as solving systems of linear equations, optimizing resources, and analyzing data. By the end of this module, you will develop the ability to use matrix algebra to solve complex problems and make informed decisions in business contexts.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Define the basic concepts of matrices.
2. Demonstrate proficiency in matrix operations while solving mathematical problems.
3. Apply matrix algebra to solve real-world business problems.
4. Evaluate the role of matrix algebra in business decision-making.

LEARNING ACTIVITIES/TASK LIST

1. Watch lecture video
2. Quiz/questions 1
3. Reading material 1: Notes
4. Quiz/ Questions
5. Reading material 2: Book chapter
6. Online Group discussion
7. Reading material 3: Review power presentation PDF notes
8. Complete end of module assessment

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning
READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment...)

Activity 1

You are required to watch/listen to the introduction video to Matrices.
The video provides a comprehensive introduction to matrices, covering their definition, order, elements, addition, scalar multiplication, and subtraction. The video explains these concepts with clear examples and step-by-step demonstrations.
After watching, make short notes for your revision and attempt Quiz/questions labeled **task 1** and discuss the answers with your peers.

VIDEO 1: Introduction to Matrices

Please click on the link below:
https://youtu.be/yRwQ7A6jVLk?si=fYPOghjrO-cSgdNb

Task 1

QUIZ/ Questions 1

1. Define a matrix and give an example of a 3x2 matrix.
2. What is the order of a matrix? How do you determine the order of a given matrix?
3. Explain the process of adding two matrices. What are the requirements for two matrices to be added?
4. How do you multiply a matrix by a scalar? Provide an example.
5. What is the difference between matrix addition and matrix multiplication? Explain the rules for each operation.

Answers

1. A matrix is a rectangular array of numbers arranged in rows and columns. Example:

[1 2 3], $[4]$, $\begin{bmatrix} 2 & 4 & 3 \\ 8 & 2 & 10 \\ 2 & 7 & 6 \end{bmatrix}$

2. The order of a matrix is the number of rows and columns it has. It is written as rows x columns. For example, the matrices above have an order of 1x3, 1x1 and 3x3 respectively.

3. To add two matrices, you must add the corresponding elements of each matrix. The resulting matrix will have the same order as the original matrices. For example, adding the following matrices:

[1 2] + [3 4] = [4 6]

4. To multiply a matrix by a scalar, you multiply each element of the matrix by the scalar. For example, multiplying the matrix [1 2] by 3 gives [3 6].

5. Matrix addition and matrix multiplication are two different operations. Matrix addition involves adding corresponding elements, while matrix multiplication involves multiplying rows of the first matrix by columns of the second matrix. The rules for matrix multiplication are more complex and involve a dot product calculation.

Activity 2

You are required to read the notes on matrix operation. Upon completion; you are to undertake **(Task 2).**

READING MATERIAL 1: Notes

Transpose of a Matrix

Recall, a matrix is a rectangular array of numbers arranged in rows and columns. Matrices are denoted by capital letters such as A, B, and C. Mathematically; a matrix can be represented as:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where a_{ij} represents the element in the i-th row and j-th column.

The transpose of a matrix A is obtained by flipping its rows into columns and vice versa.

If A is an m×n matrix, its transpose A^T is an n×m matrix.

For example:

$$A = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 2 & 7 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 2 & 8 & 2 \\ 4 & 2 & 7 \end{bmatrix}$$

Special Vectors and Matrices

1. Zero Matrix

A matrix where all elements are zero.

$$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2. Identity Matrix

   A square matrix where all diagonal elements are 1, and all off-diagonal elements are 0. Denoted as I_n for an n×n matrix.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Diagonal Matrix

   A matrix where all non-diagonal elements are zero.

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

4. Column and Row Vectors

   A matrix with a single column or row.

Column Vector=$\begin{bmatrix}1\\2\\3\end{bmatrix}$, Row Vector=$[1 \quad 2 \quad 3]$

Matrix Representation of a Linear System

A system of linear equations can be represented using matrices. For instance, consider the system:

2x+3y=5

4x−y=1

This can be written as AX=B, where:

$A = \begin{pmatrix}2 & 3\\4 & -1\end{pmatrix}, X = \begin{pmatrix}X\\Y\end{pmatrix}, B = \begin{pmatrix}5\\1\end{pmatrix}$

The solution to the system is found by solving AX=B.

Matrix Inversion and Solution to a Linear System

The inverse of a matrix A, denoted A^{-1}, is the matrix that satisfies the condition:

$A \cdot A^{-1} = I$ Where; I is an identity matrix.

If A^{-1} exists, the solution to the system AX=B can be found as:

$X = A^{-1} \cdot B$

Matrix inversion can be calculated using various methods, including the adjoint method and Gauss-Jordan elimination.

Determinants

The determinant of a square matrix provides important information about the matrix, including whether it has an inverse.

A matrix A has an inverse if and only if its determinant, denoted det(A), is non-zero.

For a 2×2 matrix:

$A = \begin{pmatrix}a & b\\c & d\end{pmatrix},$

$\det(A) = ad - bc$

For a 3×3 matrix, the determinant can be computed using cofactor expansion:

$$A = \begin{bmatrix}a & b & c\\d & e & f\\g & h & i\end{bmatrix}$$

$\det(A) = a.det\begin{pmatrix}e & f\\h & i\end{pmatrix} - b.\begin{pmatrix}d & f\\g & i\end{pmatrix} + c.\begin{pmatrix}d & e\\g & h\end{pmatrix}$

Matrix Inversion Using the Adjoint Method

To find the inverse of a matrix using the adjoint method, follow these steps:

i) Compute the cofactors for each element of the matrix.

ii) Form the cofactor matrix.

iii) Take the transpose of the cofactor matrix (this gives the adjoint).

iv) Divide the adjoint matrix by the determinant of the original matrix.

Illustration

Given matrix A:

$$A = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$$

First, compute det(A);
det(A)=(4×2)−(3×3)=8−9=−1
Next, compute the cofactor matrix and adjoint, and finally divide by det(A).

Properties of Determinants

1. Multiplicative Property: det(AB)=det(A)·det(B)
2. Determinant of a Transpose: det(A^T)=det(A).
3. Determinant of an Inverse: $\det(A^{-1}) = \frac{1}{\det(A)}$
4. Row or Column Swap: Swapping two rows or columns of a matrix changes the sign of its determinant.
5. Scalar Multiplication: If a matrix A has a scalar multiple k, then $\det(kA) = \det\,(k^n \det\,(A)$, where n is the order of the matrix.

Task 2

QUIZ/ QUESTIONS 2

6. Find the inverse of the following system of linear equations

$$20x - 500 = -6y$$
$$-200 - 2y = -10x$$

Answer: $A^{-1} = \frac{1}{-20}\begin{pmatrix} 2 & -6 \\ -10 & 20 \end{pmatrix} = \begin{pmatrix} -\frac{1}{10} & \frac{3}{10} \\ \frac{1}{2} & -1 \end{pmatrix}$

ACTIVITY 3

READING MATERIAL 2: Book Chapter

Vali, S. (2014). Principles of mathematical economics. Atlantis Press
Please read **Section 13.4** pages 445-453 of the provided material. Focus on the following key sections:
Understand the process of calculating **determinants** for matrices of order 1, 2, and 3.
Pay attention to the **Laplace expansion** method for determining higher-order matrices and how to compute minors and cofactors.
Make sure to understand each concept as it will be crucial for solving problems involving linear systems and matrix operations. Undertake **task 3**
Task 3

ONLINE DISCUSSION

Use chats, forum, question/answer, message my teacher to engage others.

Using the matrix below, explain step-by-step how the Laplace expansion method is applied to calculate the determinant of a 3x3 matrix.
Ensure you highlight key concepts such as minors, cofactors, and the choice of row or column for expansion. Include detailed calculations and reasoning for each step. Share your response in the LMS and discuss with your peers.
Matrix:

$$A = \begin{bmatrix} 3 & 5 & 3 \\ 1 & 4 & 7 \\ 6 & 8 & 9 \end{bmatrix}$$

Guiding Points:

1. Choose any row or column for your expansion.
2. Define what a minor and a cofactor are in the context of Laplace expansion.
3. Show how to calculate the determinant using the chosen row or column.
4. Reflect on whether the determinant would change if a different row or column were selected, and why.

Expected Answer: The response should demonstrate a clear understanding of the Laplace expansion method, correctly calculating the determinant and explaining the process in a way that shows comprehension of the underlying concepts.

Activity 5

READING MATERIAL 3: PowerPoint Notes

You are required to read the power point notes provided and undertake **task 4**

MATRICES METHOD-SOLVING TWO SIMULTANEOUS EQUATIONS

One of the most important applications of matrices is to the solution of linear simultaneous equations. On this chapter we explain how this can be done.

Writing simultaneous equations in matrix form

Consider the simultaneous equations;

$x + 2y = 4$

$3x - 5y = 1$

Provided you understand how matrices are multiplied together you will realize that these can be written in matrix form as;

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Writing

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}, \qquad X = \begin{pmatrix} x \\ y \end{pmatrix} and\ \ B = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

We have $\quad AX = B$

This is the matrix form of the simultaneous equations. Here the only unknown is the matrix X, since A and B are already known. A is called the matrix of coefficients.

Solving the simultaneous equations

Using Cramer's Rule to Solve Two Equations with Two Unknowns

Here we will be learning how to use Cramer's Rule to solve a linear system with two equations and two unknowns. Cramer's Rule is one of many techniques that can be used to solve systems of linear equations. Cramer's Rule involves the use of determinants to find the solution and like any other technique it has its advantages and disadvantages. Cramer's Rule itself is very simple, but the notation used requires a little bit of explanation, so let's take a look at Cramer's Rule.

$$\boldsymbol{x = \frac{D_x}{D} and\ y = \frac{D_y}{D}}$$

To help explain the notation, consider the following system of equations:

3x−2y= 17

4x+5y= −8

To find the values of x and y there are three different values that we need to calculate, they are D, Dx, and Dy. In all three cases the "D" stands for the determinant, now let's look at what they represent.

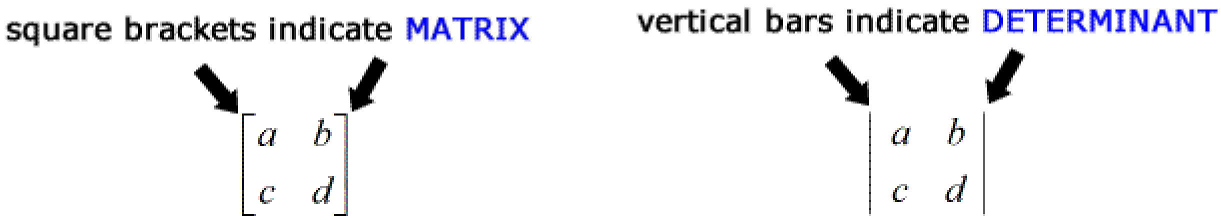
$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix}$ Here we use the x and y values from the problem to create a 2 × 2 matrix

$D_x = \begin{vmatrix} 17 & -2 \\ -8 & 5 \end{vmatrix}$ Here we replace the x-values in the first column with the values after the equal sign and leave the values in the y column unchanged.

$D_y = \begin{vmatrix} 3 & 17 \\ 4 & -8 \end{vmatrix}$ Here we replace the y-values in the second column with the values after the equal sign and leave the values in the x column unchanged.

Once we have calculated the values of D, Dx, and Dy we can apply Cramer's Rule to find x and y.

In terms of notations, a matrix is an array of numbers enclosed by square brackets while determinant is an array of numbers enclosed by two vertical bars.



The formula to find the **determinant of a 2 x 2 matrix** is very straightforward.

Let's take a quick review:

The Determinant of a 2 x 2 Matrix

$$\text{Let matrix A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

THEN

always subtract

$$\text{The determinant of matrix A is } |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

To use Cramer's Rule to solve a system of two equations with two unknowns, we need to follow these steps:

Step 1: Find the determinant, D, by using the x and y values from the problem.

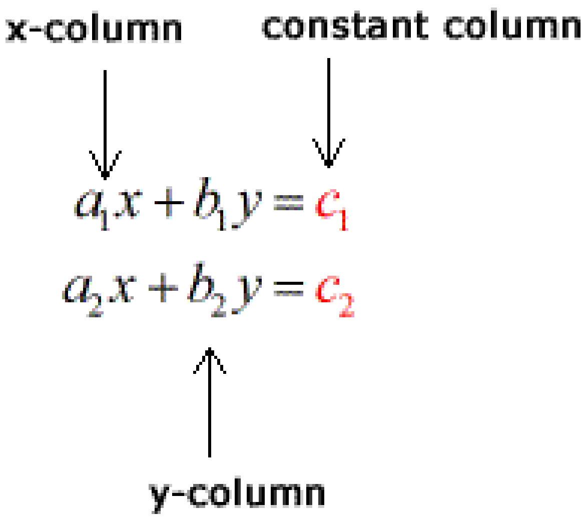
Step 2: Find the determinant, Dx, by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

Step 3: Find the determinant, Dy, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

Step 4: Use Cramer's Rule to find the values of x and y.

Cramer's Rules for Systems of Linear Equations with Two Variables

Given a linear system;



Assign names for each matrix

Coefficient matrix:

$$D = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

X – Matrix:

$$D_x = \begin{bmatrix} c_1 & b_1 \\ c_2 & b_2 \end{bmatrix}$$

Y – Matrix:

$$D_y = \begin{bmatrix} a_1 & c_1 \\ a_2 & c_2 \end{bmatrix}$$

To solve for the variable x:

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

To solve for the variable y:

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

In the example above therefore;

Example

Use Cramer's Rule to solve;

$$3x - 2y = 17$$

$$4x + 5y = -8$$

Step 1: Find the determinant, D, by using the x and y values from the problem.

$$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 15 - (-8) = 23$$

Step 2: Find the determinant, D_x by replacing the x-values in the first column with the values after the equal sign leaving the y column unchanged.

$$D_x = \begin{vmatrix} 17 & -2 \\ -8 & 5 \end{vmatrix} = 85 - 16 = 69$$

Step 3: Find the determinant ,D_y, by replacing the y-values in the second column with the values after the equal sign leaving the x column unchanged.

$$D_y = \begin{vmatrix} 3 & 17 \\ 4 & -8 \end{vmatrix} = -24 - 68 = -92$$

Step 4: use Cramer's Rule to find the values of x and y

$$x = \frac{D_x}{D} = \frac{69}{23} = 3$$

$$y = \frac{D_y}{D} = \frac{-92}{23} = -4$$

The answer written as an ordered $(3, -4)$.

Matrix inverse method

Given AX=B we can multiply both sides by the inverse of A, provided this exists, to give $A^{-1}AX=A^{-1}B$

But $A^{-1}A=I$, the identity matrix. Furthermore, IX=X, because multiplying any matrix by an identity matrix of the appropriate size leaves the matrix unaltered. So

$X = A^{-1}B$

If AX=B, then $X=A^{-1}B$

This result gives us a method for solving simultaneous equations. All we need to do is to write them in matrix form, calculate the inverse of the matrix of coefficients, and finally perform a matrix multiplication.

Example.

Solve the simultaneous equations

x+ 2y= 4

3x−5y= 1

Solution

We have already seen these equations in matrix form:

$$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

We need to calculate the inverse of $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$.

$$A^{-1} = \frac{1}{(1)(-5) - (2)(3)}\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} = -\frac{1}{11}\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}$$

$$X = A^{-1}B = -\frac{1}{11}\begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= -\frac{1}{11}\begin{pmatrix} -22 \\ -11 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Hence $x = 2, y = 1$ is the solution of the simultaneous equation

Task 4

MASTERY EXERCISE 1.1

1. Consider a factory that produces two products, A and B. Each product requires labor and raw materials, and the available resources are limited. The production requirements are as follows:

   Product A: 3 units of labor, 4 units of raw materials

   Product B: 2 units of labor, 5 units of raw materials

The factory has 12 units of labor and 20 units of raw materials available.

Required

a) Represent the information as a matrix equation

b) Using matrices, calculate the approximate units of product A and B the factory can produce B while staying within resource limits.

Answers:

a. $\begin{pmatrix} 3 & 2 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} X_A \\ X_B \end{pmatrix} = \begin{pmatrix} 12 \\ 10 \end{pmatrix}$

b. Approximately 2.86 units of product A and 1.71 units of product B

7. A furniture company produces three types of chairs: A, B, and C. The sales of these chairs in two cities, City A and City B, are as follows:
   - In City A: 120 chairs of type A, 150 chairs of type B, and 100 chairs of type C were sold.
   - In City B: 90 chairs of type A, 130 chairs of type B, and 150 chairs of type C were sold.

Required

If the cost of each chair is \$30, \$20 and \$10 and the selling price of each chair is \$35, \$30 and \$20 respectively. Using matrices and matrix operations;

a. Find the total cost of the factory for the total sales made
b. Find the total profit of the factory

Answers:

a. Total cost = $\begin{bmatrix} 120 & 150 & 103 \\ 90 & 130 & 150 \end{bmatrix} \times \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 7600 \\ 6800 \end{bmatrix}$

   Therefore total cost is 7600+6800=**\$14,400**

b. Total Revenue =$\begin{bmatrix} 120 & 150 & 100 \\ 90 & 130 & 150 \end{bmatrix} \times \begin{bmatrix} 35 \\ 30 \\ 20 \end{bmatrix} = \begin{bmatrix} 10700 \\ 10050 \end{bmatrix}$

   Therefore total revenue is 10700+10050**=\$20,750.**

Task 5

END OF MODULE ASSESSMENT 1.2

1. Solve the following system of linear equations using matrix inversion:

   2x+3y=5

   4x+y=11

   Express the system in the form AX=B, find A^{-1}, and calculate X.

   Answer: x=2 and y=−1

2. Define the determinant of a 3x3 matrix and calculate the determinant of the following matrix using cofactor expansion:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

   Answer: det(A)=31

3. Explain the conditions under which a square matrix has an inverse. What is the significance of the determinant in determining the invertibility of a matrix?

Short Answer:

A square matrix has an inverse if and only if its determinant is non-zero. The determinant provides critical information: if det(A)=0, the matrix is singular and does not have an inverse. If det(A)≠0, the matrix is invertible. The inverse matrix is useful in solving systems of linear equations.

4. Solve the system of equations using Cramer's Rule:

   $3x-2y=17$

   $4x+5y=-8$

   Answer: $x=3$and $y=-4$.

5. Given the matrices:

   $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

   Find the product A×B

   Answer: $A \times B = \begin{bmatrix} 30 & 24 & 18 \\ 84 & 69 & 54 \\ 138 & 114 & 90 \end{bmatrix}$

REFERENCE LIST AND FURTHER READING

1. BBP Learning Media. (2012). *CIMA Fundamentals of Business Mathematics Revision Kit* (Vol. 00003). BPP Learning Media. https://research.ebsco.com/linkprocessor/plink?id=df1c1006-5fc3-3cd2-8fdc-70aa8c0d1872
2. Bradley, T. (2013). *Essential mathematics for economics and business*. John Wiley & Sons.
3. Clendenen, G., & Salzman, S. A. (2018). *Business mathematics*. Pearson.
4. Gary Bronson, Richard Bronson, & Maureen Kieff. (2021). *Mathematics for Business*. Mercury Learning and Information. https://research.ebsco.com/linkprocessor/plink?id=0b9336e0-e2fc-3560-9c10-c5c674163b70
5. M. S. Mukras : (2004), *Elements of Mathematical Economics*, KLB, Nairobi
6. The Organic Chemistry Tutor. (2018). Intro to Matrices. In *YouTube*. https://www.youtube.com/watch?v=yRwQ7A6jVLk
7. Ummer, E. K. (2012). *Basic mathematics for economics, business, and finance*. Routledge.
8. Vali, S. (2014). Principles of mathematical economics. Atlantis Press (pg 438-453)

TAKE HOME MESSAGE

Congratulations for successfully completing the module! Take a moment to reflect on the powerful analytical skills you've acquired so far. What has been your learning experience throughout this module?

WHAT'S NEXT?

- Module 4 Review Assignments
- Module 5: Optimization problems in Business
- As we transition into the exciting realm of **Optimization Problems in Business**, prepare to explore powerful tools that help companies maximize profits and minimize costs. This module will empower you with the knowledge to tackle resource allocation challenges, providing insights that are critical for real-world business success.

  With your growing expertise in mathematical applications, you're on track to becoming a strategic decision-maker.

  Embrace this journey with enthusiasm, as it will open doors to innovative problem-solving techniques in the dynamic world of business. Let's dive in!

MODULE 5: OPTIMIZATION PROBLEMS IN BUSINESS

INSTRUCTIONAL HOURS: 3

MODULE OVERVIEW

In this module, you will explore the fundamental principles of optimization, which are crucial for making efficient and effective business decisions. We will begin with an introduction to basic optimization problems, laying the foundation for understanding how to achieve the best outcomes under given constraints. You will learn about linear programming, a powerful method used to solve optimization problems, and how to formulate linear programming models to represent real-world business scenarios. The module will also cover the graphical solution method, enabling you to visualize and solve optimization problems with two variables. Finally, we will focus on practical applications of these techniques in resource allocation, profit maximization, and cost minimization. By the end of this module, you will have the skills needed to apply optimization methods to enhance decision-making and improve business performance.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Describe the basic optimization problems and the key concepts and terminologies used in linear programming and optimization.
2. Formulate and construct linear programming models for various business scenarios.
3. Solve and interpret optimization problems using the graphical solution method.
4. Evaluate the role of optimization in business decision-making.

LEARNING ACTIVITIES/TASK LIST

1. listen to a pre-recorded lecture
2. Quiz/questions 1
3. Watch Lecture Video
4. Masterly exercise 1.1
5. Reading material 1: Book chapter
6. Quiz/questions 1
7. Online discussion
8. End of module assessment 1.1

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning
READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment…)

Activity 1

You are required to watch/listen to the Introduction to Basic Optimization Problems video/powerpoint (to be developed)

Upon completion; undertake the quiz or questions labeled **(Task 1).**

Introduction to Basic Optimization Problems

An optimization problem is a mathematical framework used to find the best solution, subject to certain constraints, by maximizing or minimizing an objective function. The "best" solution could mean minimizing costs, maximizing profits, or achieving a specific goal efficiently (Hillier and Lieberman, 2012).

Optimization problems in business involve finding the best possible solution to achieve a specific objective, such as maximizing profit or minimizing cost, while satisfying given constraints.

These problems typically require mathematical models to analyze and solve real-world business issues efficiently.

Understanding optimization is essential in business as it allows decision-makers to allocate resources optimally, improve efficiency, and solve complex problems involving trade-offs between various objectives. It provides systematic approaches to make informed, data-driven decisions across different sectors like production, finance, marketing, and logistics.

History of Optimization Problems

Optimization has its roots in mathematics, starting from ancient times when scholars like Euclid (circa 300 BCE) studied geometric problems involving maximizing and minimizing lengths.

In modern times, optimization theory developed with the advent of calculus, initiated by Sir Isaac Newton and Gottfried Wilhelm Leibniz in the 17th century.

Later, in the 19th and early 20th centuries, the field of optimization grew with contributions from mathematicians like Joseph-Louis Lagrange and Karl Weierstrass, who introduced methods such as Lagrange multipliers for constrained optimization. The foundation of Linear Programming (LP), a specific area of optimization, was laid by Leonid Kantorovich and George Dantzig in the 1940s.

Dantzig's Simplex Method, introduced in 1947, revolutionized the solution of linear optimization problems, becoming widely used in economics, business, and engineering.

The development of optimization methods continued through the 20th century with advancements in non-linear optimization, integer programming, and dynamic programming.

Types of Optimization Problems

1. **Maximization Problems**: These focus on increasing an objective function, like profits or output.
2. **Minimization Problems**: These focus on reducing aspects such as costs, waste, or time.
3. **Resource Allocation Problems**: These involve optimizing the use of limited resources, like budget, labor, or materials, to achieve the best possible outcome.

Elements of an Optimization Problem

1. **Objective Function**: The mathematical function that represents the goal of the optimization, such as total profit or cost.
2. **Constraints**: The limitations or restrictions on resources, such as budget limits, production capacity, or time.
3. **Decision Variables**: The variables representing quantities to be determined, such as how many units of a product to produce or how much money to allocate to different departments.

Formulating Optimization Problems

In business scenarios, optimization problems are typically formulated as:

1. Objective function:

$$Maximize(Profit) = 35x_1 + 20x_2$$

where $x_1$and x_2 represent decision variables (number of units of different products)

2. Constraints:

$x_1 + 2x_2 \leq 100$ Resource constraint 1

$2x_1 + x_2 \leq 80$ Resource constraint 2

$x_1, x_2 100 \geq 0$ Non negativity constraint

These constraints reflect available resources like labor or raw materials.

Business Applications of Optimization

In business, optimization is used to make better decisions by maximizing or minimizing key objectives, such as profit, cost, or efficiency. Some key applications include:

i) Production Optimization: Companies optimize production processes to maximize output while minimizing cost. This includes determining the optimal use of raw materials, labor, and machinery to achieve the best possible production levels.

ii) Supply Chain Management: Businesses use optimization techniques to improve logistics, such as determining the most efficient delivery routes, minimizing shipping costs, or optimizing inventory levels to meet demand while minimizing storage costs.

iii) Marketing Optimization: Companies optimize marketing strategies, such as maximizing return on investment (ROI) in advertising campaigns or pricing strategies by analyzing consumer demand and pricing elasticity.

iv) Financial Portfolio Optimization: In finance, optimization is used to create portfolios that maximize return for a given level of risk, often referred to as the Markowitz Portfolio Theory.

v) Profit Maximization: Firms use optimization models to determine how many units of a product should be produced and sold to maximize profit, given production costs, market demand, and other constraints.

Methods of Solving Optimization Problems

1. **Graphical Method**: Used for simple two-variable problems where the objective function and constraints are plotted on a graph to find the optimal point.
2. **Linear Programming (LP)**: A mathematical method for determining the best outcome, particularly useful for larger and more complex problems.
3. **Simplex Method**: An algorithm used to solve linear programming problems that involve multiple variables and constraint.

From the foregoing discussion it is clear that the history and application of optimization highlight its critical role in decision-making, especially in today's data-driven business environment, where it is essential for maximizing efficiency and profitability.

Task 1

QUIZ/ QUESTIONS 1

1. What is an optimization problem?

a) A problem focused on minimizing or maximizing an objective function.

b) A mathematical model with no constraints.

c) A decision-making process that disregards trade-offs.

d) A method to solve only financial issues.

Answer: a)

2. Who introduced the Simplex Method in 1947 for solving linear optimization problems?

a) Isaac Newton

b) George Dantzig

c) Euclid

d) Karl Weierstrass

Answer: b)

3. Which of the following is a type of optimization problem in business?

a) Resource Allocation Problem

b) Cost Expansion Problem

c) Non-resource Problem

d) Product Minimization Problem

Answer: a) Resource Allocation Problem

4. In optimization, the mathematical function representing the goal is called the:

a) Constraint

b) Decision Variable

c) Objective Function

d) Profit Equation

Answer: c)

5. Which of the following methods is used for solving simple two-variable optimization problems graphically?

a) Linear Programming

b) Dynamic Programming

c) Graphical Method

d) Simplex Method

Answer: c) Graphical Method

6. Maximization problems in business aim to reduce costs and improve efficiency.

Answer: False (Maximization problems aim to increase the objective function, such as profit.)

7. Lagrange multipliers are used for solving unconstrained optimization problems.

Answer: False (Lagrange multipliers are used for constrained optimization problems.)

8. Optimization methods can only be applied in production processes.

Answer: False (Optimization methods are applied in various sectors such as finance, marketing, and logistics.)

9. The Simplex Method is an algorithm used in solving linear programming problems.

Answer: True

10. Objective functions in business optimization can either maximize or minimize specific outcomes, such as profit or cost.

Answer: True

Activity 2

You are required to watch the video on Formulation of Linear Programming Problem here, which explains key components such as the objective function, constraints, and the importance of non-negativity conditions.
After watching the video, be prepared to answer mastery exercise labeled **task 2** and discuss the answers with your peers.

VIDEO 1: Formulation of Linear Programming Problems

Please click on the link below:
https://youtu.be/uJFiR0DG2Bw?si=_ayFxbHVZBLkFotM

Task 2

MASTERY EXERCISE 1.1

1. A local farmer has land for planting corn and soybeans. Each acre of corn planted requires 2 hours of labor and yields a profit of $50. Each acre of soybeans requires 1 hour of labor and yields a profit of $40. The farmer has 100 hours of labor available. How can you formulate this scenario as a linear programming model (including the objective function and constraints)?

2. A bakery sells cookies and muffins. Each cookie requires 3 units of flour and 2 units of sugar, while each muffin needs 1 unit of flour and 4 units of sugar. They have 60 units of flour and 80 units of sugar available. They want to maximize their daily profit, with a profit of $2 per cookie and $3 per muffin. Formulate this problem as a linear programming model.
3. Imagine you're a designer creating t-shirts and hoodies. Each t-shirt requires 1 meter of fabric and 2 hours of stitching, while each hoodie requires 2 meters of fabric and 3 hours of stitching. You have 100 meters of fabric and 120 hours of stitching available. You earn a profit of $10 per t-shirt and $20 per hoodie. How can you model this scenario as a linear programming problem?

Answers

Problem 1: Farmer's Dilemma

Objective Function:

Maximize Profit: $P = 50x + 40y$

Constraints:

Labor constraint: $2x + y \leq 100$

Non-negativity constraints: $x \geq 0$, $y \geq 0$

Problem 2: Bakery Production

Objective Function:

Maximize Profit: $P = 2x + 3y$

Constraints:

Flour constraint: $3x + y \leq 60$

Sugar constraint: $2x + 4y \leq 80$

Non-negativity constraints: $x \geq 0$, $y \geq 0$

Problem 3: Designer's Dilemma

Objective Function:

Maximize Profit: $P = 10x + 20y$

Constraints:

Fabric constraint: $x + 2y \leq 100$

Stitching constraint: $2x + 3y \leq 120$

Non-negativity constraints: $x \geq 0$, $y \geq 0$

Activity 3

READING MATERIAL 1: BOOK CHAPTER

Chapter 3: Hillier, F. S., & Lieberman, G. J. (2015). *Introduction to operations research* (page 26-30)

Please read Chapter 3, pages 26-30 of *Introduction to Operations Research* by Hillier, F. S., & Lieberman, G. J. (2015), which covers the formulation of Linear Programming Problems and their Graphical Solutions. After reading, proceed to the quiz labeled **"Task 3"** to apply the concepts you have learned.

Task 3

QUIZ/ QUESTIONS 2

1. Use the graphical method to solve this problem:

Maximize $Z = 15x_1 + 20x_2$

subject to:

$x_1 + 2x_2 \leq 10$

$2x_1 + 3x_2 \leq 6$

$x_1 + x_2 \leq 6$

$x_1 \geq 0,\ x_2 \geq 0$

Answers: The maximum value of Z is 100, occurring at $x_1 = 4$ and $x_2 = 2$.

2. Consider the following model:

   Minimize $Z = 40x_1 + 50x_2$

   Subject to:

   $2x_1 + 3x_2 \leq 30$

   $x_1 + x_2 \leq 12$

   $2x_1 + x_2 \leq 20$

   and

   $x_1 \geq 0,\ x_2 \geq 0$

   Required

   (a) Use the graphical method to solve this model.

   (b) How does the optimal solution change if the objective function is changed to $Z = 40x_1 + 70x_2$?

   (c) How does the optimal solution change if the third functional constraint is changed to $2x_1 + x_2 \leq 15$?

   Answers

   a. The minimum value of Z is 400, occurring at $x_1 = 10$ and $x_2 = 0$.
   b. The optimal solution remains $x_1 = 10$ and $x_2 = 0$, but the minimum value of Z has changed to 400. This indicates that the change in the objective function coefficient did not affect the optimal decision variables but did impact the overall objective function value.
   c. When the third constraint is changed to $2x_1 + x_2 \leq 15$, the optimal solution becomes $(x_1, x_2) = (3.75, 7.5)$, and the minimum value of Z is 525.

3. Consider the following problem, where the value of c_1 has not yet been ascertained.

Maximize $Z = c_1x_1 + 2x_2$

subject to:

$4x_1 + x_2 \leq 12$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Required

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of c_1.

Answers

If $c_1 < 0$: The optimal solution is (0, 12) and $Z = 24$.

If $0 \leq c_1 \leq 8$: The optimal solution is (2.8, 0.8) and $Z = 2.8c_1 + 1.6$.

If $c_1 > 8$: The optimal solution is (2, 0) and $Z = 2c_1$.

Activity 4

ONLINE DISCUSSION

Use chats, forum, question/answer, message my teacher to engage others.

Consider the following practice questions designed to illustrate the application of linear programming in business, particularly focusing on **profit maximization** and **cost minimization**. You are required to discuss with your peers, evaluate these questions following the procedure outlined below and share your answers on the LMS.

Procedure for Evaluating the questions

1. **Understand the Objective**:
    - For profit maximization problems, identify the **objective function** that represents the total profit.
    - For cost minimization problems, identify the **objective function** that represents the total cost.
2. **Identify the Decision Variables**:

   - Determine the key variables involved in the business problem (e.g., the number of products to produce or the amount of resources to allocate).
3. **Formulate the Constraints**:
   - Translate any limitations or restrictions in the problem into **inequality constraints**. These could relate to resources like labor, materials, time, or budgets.
4. **Graph the Constraints** (for graphical method):
   - Plot each constraint as a line or curve on a graph.
   - Identify the feasible region that satisfies all constraints.
5. **Determine the Corner Points**:
   - Identify the intersection points of the constraints, which form the boundary of the feasible region. These points are where the objective function can potentially reach its maximum or minimum.
6. **Evaluate the Objective Function**:
   - Calculate the value of the objective function at each corner point or vertex.
   - The optimal solution will occur at one of these points.
7. **Interpret the Results**:
   - Provide a clear interpretation of what the optimal solution means in the context of the business problem (e.g., how many units to produce or resources to allocate).

Questions 1: Profit Maximization

A company produces two products, X and Y:

- The profit per unit of X is $40, and the profit per unit of Y is $60.
- To produce one unit of X, it requires 2 hours of labor and 3 units of raw material.
- To produce one unit of Y, it requires 4 hours of labor and 2 units of raw material.
- The company has 80 hours of labor and 60 units of raw material available per week.

Formulate and solve a linear programming model to determine how many units of each product the company should produce to maximize profit.

Question 2: Cost Minimization

A factory needs to produce two types of products, A and B:

- The production cost per unit of A is $20, and for B, it is $30.
- Each unit of A requires 3 hours of machine time and 2 hours of labor.
- Each unit of B requires 4 hours of machine time and 1 hour of labor.
- The factory has 120 hours of machine time and 90 hours of labor available.

Formulate and solve a linear programming model to determine how many units of each product the factory should produce to minimize total production cost.

Task 5

END OF MODULE ASSESSMENT 1.2

1. Solve the following linear programming problem using the graphical method:

   Maximize $Z=30x_1+50x_2$

   Subject to:

   $2x_1+4x_2\leq 16$

   $x_1+3x_2\leq 9$

   $x_1\geq 0,\ x_2\geq 0$

   Answer: The maximum value of $Z=260$ occurs at $x_1=4,\ x_2=2$

2. A factory manufactures two products, X and Y. The cost to produce one unit of X is $25, and for Y, it's $35. The factory has a budget of $2000. It takes 3 hours of labor to produce one unit of X and 2 hours for one unit of Y, and a maximum of 180 labor hours is available. Formulate a linear programming model to minimize costs.

   Answers:

   Objective Function:

   Minimize Cost: $Z=25X+35Y$

   Constraints:

- Budget: $25X+35Y\leq 2000$
- Labor: $3X+2Y\leq 180$
- Non-negativity: $X\geq 0,\ Y\geq 0X$

3. A company produces two products, A and B. The profit per unit for A is \$50, and for B is \$70. Each product requires labor and material. Product A needs 2 hours of labor and 3 units of material, while product B needs 4 hours of labor and 2 units of material. The company has 120 hours of labor and 100 units of material available. Formulate the linear programming model to maximize profit.

   Answers:

   Objective Function:

   Maximize Profit: $Z=50A+70$

   Constraints:

   Labor: $2A+4B\leq 120$

   Material: $3A+2B\leq 100$

   Non-negativity: $A\geq 0$, $B\geq 0A$

REFERENCE LIST AND FURTHER READING

1. Carter, M., Price, C. C., & Rabadi, G. (2018). *Operations research: a practical introduction*. Chapman and Hall/CRC.
2. Dantzig, G. B. (1951). Maximization of a linear function of variables subject to linear inequalities. *Activity analysis of production and allocation*, *13*, 339-347.
3. Hillier, F. S., & Lieberman, G. J. (2015). *Introduction to operations research*. McGraw-Hill.
4. *Infinity Learn NEET*. (2018). Linear Programming - Formulation 1 | Don't Memorise [YouTube Video]. In *YouTube*. https://www.youtube.com/watch?v=uJFiR0DG2Bw
5. Taha, H. A. (2013). *Operations research: an introduction*. Pearson Education India.

TAKE HOME MESSAGE

After covering the module on Optimization Problems in Business successfully, reflect on this: How can your newfound understanding of optimization techniques help you make more informed decisions in resource allocation, cost management, and profit maximization to enhance business efficiency and sustainability?

WHAT'S NEXT?

- Module 5 Review Assignment
- Module 6: Mathematics of Finance
- Get ready for Module 6, where we'll dive into the fascinating world of Mathematics of Finance! Prepare to explore how concepts like interest rates, present and future values, annuities, and investment strategies can shape financial decisions in business. This journey will equip you with the tools to make sound financial choices, paving the way for smart investments and sustainable growth.

MODULE 6: MATHEMATICS OF FINANCE

INSTRUCTIONAL HOURS: 4

MODULE OVERVIEW

In this module, you will delve into the mathematical principles that underpin financial decision-making. We will start with the basics of simple and compound interest, exploring how these concepts apply to various financial products and investments. You will learn to calculate the present and future value of money, understanding how time and interest rates affect the value of financial assets. The module will also cover annuities and perpetuities, focusing on how these financial instruments generate regular payments over time. Additionally, you will explore loan amortization methods and investment appraisal techniques, equipping you with the tools to assess and manage financial decisions effectively. By the end of this module, you will be proficient in applying mathematical concepts to solve real-world financial problems and make informed investment choices

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Identify and define key financial concepts.
2. Demonstrate proficiency in computing simple and compound interest, present and future values, as well as evaluating annuities and perpetuities.
3. Apply mathematical techniques to solve problems related to loan amortization and investment appraisal to make informed financial decisions.
4. Evaluate the impact of financial mathematics on personal and business financial planning and analysis in making sound financial decisions.

LEARNING ACTIVITIES/TASK LIST

1. Listen to a pre-recorded lecture
2. Undertake the quiz or questions1
3. Watch video 1
4. Mastery Exercise 1.1
5. Read material 1: Notes
6. Engage in online group discussion 1
7. Watch video 2
8. Undertake the quiz or questions 2
9. End of Module Assessment 1.1

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning
READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment…)

Activity 1
You are required to listen to the prerecorded lecture on the introduction to Mathematics of Finance.
Upon completion; undertake the quiz or questions **(Task 1)**

Introduction to Mathematics of Finance

Mathematics of finance is a specialized field that applies mathematical concepts and techniques to financial problems. It involves the use of various mathematical tools, such as algebra, calculus, statistics, and probability theory, to analyze and solve financial problems(Steland, 2012).
Mathematics of Finance plays a pivotal role in business and economics, offering the mathematical framework to analyze, understand, and optimize financial transactions. This module introduces key financial principles and mathematical techniques that help in making sound decisions related to investments, loans, savings, and other financial instruments.
Understanding the time value of money, interest rates, and investment appraisal is critical for anyone looking to manage personal or business finances effectively.
In the financial world, both individuals and businesses regularly face questions such as:

- How much should I invest today to reach a future financial goal?
- What will be the total cost of borrowing money over time?
- Which investment options offer the best return for a given level of risk?
- How do I manage loan payments effectively?

This module will address these questions by exploring essential concepts, formulas, and applications related to finance.

Key Concepts in Mathematics of Finance

1. Time Value of Money: This is the fundamental concept that money has different values at different points in time due to factors such as interest rates and inflation.
2. Interest: This is the cost of borrowing money or the return on invested money. It can be simple or compound.
3. Present Value: The current value of a future amount of money, discounted at a specific interest rate.
4. Future Value: The value of an investment at a future date, taking into account compound interest.

5. Annuities: A series of equal payments made at regular intervals.
6. Loans and Amortization: The process of paying off a loan over time with regular payments.
7. Bonds: Debt securities that promise to pay a fixed or variable interest rate over a specified period.
8. Stocks: Equity securities that represent ownership in a company.
9. Derivatives: Financial instruments whose value is derived from the value of an underlying asset.

Key Topics Covered in This Module

1. Simple and Compound Interest

Simple Interest: Calculating interest based only on the initial principal.

Compound Interest: Calculating interest on both the initial principal and the accumulated interest from previous periods.

Understanding how different interest rates and compounding frequencies affect the growth of investments or the cost of loans.

2. Present Value (PV) and Future Value (FV) Calculations

Present Value (PV): The value today of a sum of money that is to be received or paid in the future.

Future Value (FV): The value of a sum of money at a specified date in the future, given a certain interest rate.

These concepts are fundamental in determining the time value of money, which is crucial for comparing investment options and making long-term financial decisions.

3. Annuities and Perpetuities

Annuities: A series of equal payments made at regular intervals over a specified period (e.g., loan repayments or retirement plans).

Perpetuities: A type of annuity that lasts forever, with regular payments continuing indefinitely.

These tools are used for retirement planning, loan structuring, and other financial scenarios that involve periodic payments.

4. Loan Amortization

Loan Amortization: The process of gradually paying off a loan through regular payments, where each payment covers both interest and principal.

Understanding amortization schedules helps in managing mortgages, personal loans, and other forms of long-term debt.

5. Investment Appraisal

Net Present Value (NPV) and Internal Rate of Return (IRR): Methods used to evaluate the profitability of an investment by comparing the present value of cash inflows to the initial investment cost.

These techniques allow businesses to assess which projects or investments are worth pursuing, leading to more strategic and profitable decisions.

Mathematical Tools Used in Finance

a) Algebra: Used for solving equations and inequalities involving financial variables.
b) Calculus: Used for calculating rates of change, optimization, and present value calculations.
c) Statistics: Used for analyzing financial data and making predictions.
d) Probability Theory: Used for modeling risk and uncertainty in financial markets.

Applications of Mathematics of Finance

The ability to understand and apply the concepts of Mathematics of Finance is essential for anyone involved in financial planning, whether at a personal or business level. These concepts can be applied in:

i) Personal Finance: Budgeting, savings, investments, and retirement planning.
ii) Corporate Finance: Capital budgeting, financial forecasting, risk management, and valuation.
iii) Investment Banking: Portfolio management, mergers and acquisitions, and derivatives pricing.
iv) Insurance: Premium calculation, risk assessment, and reserve management.

In conclusion therefore, you will note from the foregoing discussion that by mastering the principles in this module, you will be better equipped to make decisions that impact your financial future and the long-term success of your business.

Task 1

QUIZ/ QUESTIONS

1. Which of the following statements best defines the *time value of money*?
A) Money today is worth less than money in the future because of inflation.
B) The value of money remains constant over time if interest rates are low.
C) Money today is worth more than money in the future due to its earning potential.
D) Future money can be worth more than today's money only when inflation is zero.
Answer: C)

2. Which of the following statements is true regarding the difference between *simple interest* and *compound interest*?
A) Simple interest is calculated on the initial principal, while compound interest is calculated on the principal plus accumulated interest.
B) Simple interest grows faster than compound interest in the long term.
C) Compound interest is always higher than simple interest, regardless of the time period.
D) Simple interest is calculated annually, and compound interest is calculated monthly.
Answer: A)
3. Present Value (PV) is:
A) The future value of a sum of money adjusted for interest over time.
B) The value today of a future sum of money, discounted at a specific interest rate.
C) The value of an investment after interest has been compounded over a period.
D) The total amount received from an investment in the future.
Answer: B)
4. Loan amortization refers to:
A) The process of paying only interest on a loan over its lifetime.
B) The process of gradually paying off a loan with regular payments, where each payment includes both principal and interest.
C) The practice of repaying the loan principal in one lump sum at the end of the loan term.
D) The reduction of a loan balance solely through interest payments.
Answer: B)
5. Which of the following is an example of an *annuity*?
A) A one-time payment received at the end of 5 years.
B) An infinite series of payments made regularly over time.
C) Regular monthly payments made over 20 years for a mortgage loan.
D) A single payment that increases by a fixed amount every year.
Answer: C)

Activity 2
Watch the video below that explains the difference between simple interest and compound interest, covering the formulas for each and demonstrating how to calculate the future value of an investment using both methods. The video also provides guidance on comparing different investment options by examining their interest rates and compounding periods, helping you understand how these factors impact the overall growth of an investment.
After watching the video, proceed to **Task 2**, where you will apply the concepts discussed in the video to solve practical financial problems.

VIDEO 1: Simple and Compound Interest

Please click on the link below:
https://youtu.be/E7ZqkxlS2qM?si=Swxv_4Yz0eAxsmZN

Task 2

MASTERY EXERCISE 1.1

1. A company plans to invest $10,000 in a fixed deposit account that offers a simple interest rate of 5% per year.

   - How much interest will the company earn after 4 years?
   - What will be the total value of the investment after 4 years?

Answer: Interest = Principal × Rate × Time = $10,000 × 5% × 4 = $2,000
Total value = Principal + Interest = $10,000 + $2,000 = $12,000

2. Which investment option will yield a higher return: $1,000 invested at 8% simple interest for 5 years or $1,000 invested at 6% compounded annually for 5 years?

   Answer: $1,000 invested at 6% compounded annually for 5 years will yield a higher return.
3. You want to double your investment of $5,000. If you can earn an annual interest rate of 8%, how long will it take to double your money using compound interest?
   Answer: Approximately 9 years.

Activity 3

READING MATERIAL 1: NOTES

You are required to read the notes on Present and Future Value Calculations

After completion, you are to undertake group discussion labeled **task 3.**

Introduction to Present and Future Value (PV and FV)

In finance, Present Value (PV) and Future Value (FV) are critical concepts that are used to assess the value of money over time. The idea stems from the time value of money (TVM), which states that a sum of money today is worth more than the same sum in the future due to its potential earning capacity.

The Present Value is the current worth of a sum of money that is to be received or paid at a future date, discounted at a specific interest rate. In contrast, the Future Value is the amount of money that an investment made today will grow into at a future date, based on a certain interest rate.

1. Future Value (FV)

The **Future Value (FV)** is the amount an investment will grow to after earning interest over a specific period. It can be calculated using either **simple interest** or **compound interest**.

A. Future Value with Simple Interest

Simple interest is calculated on the original principal for each period. The formula for calculating the Future Value using simple interest is:

$FV=P\,(1+rt)$

Where:

- **FV** = Future Value
- **P** = Principal (the initial investment or loan amount)
- **r** = Annual interest rate (in decimal form)
- **t** = Time in years

Illustration (Simple Interest):

Suppose you invest $1,000 at an annual interest rate of 5% for 3 years. What will be the Future Value?

Using the simple interest formula:

$FV=1000\times(1+0.05\times3)=1000\times(1+0.15)=1000\times1.15=1150$

So, the Future Value is $1,150.

B. Future Value with Compound Interest

In compound interest, interest is calculated on the initial principal and also on the accumulated interest of previous periods. The formula for calculating the Future Value using compound interest is:

$$FV = P(1+\frac{r}{n})^{nt}$$

Where:

n = Number of compounding periods per year

All other symbols remain the same.

Illustration (Compound Interest):

Suppose you invest $1,000 at an annual interest rate of 5%, compounded quarterly (4 times a year) for 3 years. What will be the Future Value?

Using the compound interest formula:

$$FV = 1000 \times (1 + \frac{0.05}{4})^{4\times 3}$$

$$FV = 1000 \times (1 + 0.0125)^{12} = 1000 \times 1.1616 = 1161.60$$

So, the Future Value is $1,161.60.

2. Present Value (PV)

The **Present Value (PV)** is the current value of a sum of money that is to be received or paid at a future date, discounted at a certain interest rate. It answers the question: "How much is a future sum worth today?"

The formula for calculating Present Value is:

$$PV = \frac{FV}{(1 + \frac{r}{n})^{nt}}$$

Where:

- PV = Present Value
- FV = Future Value
- r = Annual interest rate (in decimal form)
- t = Time in years
- n = Number of compounding periods per year

Illustration (Present Value):

You want to have $5,000 in 4 years, and the annual interest rate is 6%, compounded annually. What is the Present Value of this future sum?

Using the present value formula:

$$PV = \frac{FV}{(1 + \frac{r}{n})^{nt}}$$

$$PV = \frac{5000}{(1 + \frac{0.06}{1})^{1\times 4}} = \frac{5000}{(1 + 0.06)^4} = \frac{5000}{1.262476} = 3960.18$$

So, the Present Value is $3,960.18.

3. Applications of Present and Future Value in Finance

1. **Investment Decisions**: Investors use FV to determine how much an investment made today will grow over time, which helps them choose between different investment opportunities.
2. **Loan Payments**: PV is used to determine the value today of future loan payments, allowing borrowers and lenders to agree on fair terms.
3. **Retirement Planning**: Individuals calculate how much they need to save today (PV) to achieve a desired retirement fund (FV).
4. **Bond Pricing**: The present value of future cash flows from a bond is used to determine its current price.
5. **Valuation of Businesses**: In mergers and acquisitions, the PV of future cash flows from the business is used to assess its value.

4. Understanding Annuities and Perpetuities

Annuities are a series of equal payments made at regular intervals over a specified period. Annuities can be evaluated using PV and FV formulas that account for each payment.

- **Future Value of an Annuity**: This calculates the total value of a series of payments at the end of the period.

$$FVannuity = P \times \left[\frac{(1+r)^t - 1}{r}\right]$$

- **Present Value of an Annuity**: This calculates the current worth of a series of future payments.

$$PVannuity = P \times \left[\frac{1-(1+r)^{-t}}{r}\right]$$

- **Perpetuities** are similar to annuities, but the payments continue indefinitely. The present value of a perpetuity is:

$$PVperpetuity = \frac{P}{r}$$

Practical Illustration of Annuities

Suppose you are offered a retirement plan where you will receive $1,000 per year for 20 years with an annual interest rate of 5%. What is the present value of this annuity?

Using the Present Value of an Annuity formula:

$$PV = 1000 \times \left[\frac{1-(1+0.05)^{-20}}{0.05}\right]$$

$$PV = 1000 \times \left[\frac{1-(1.05)^{-20}}{0.05}\right] = 1000 \times 12.4622 = 12{,}462.20$$

So, the Present Value of the annuity is $12,462.20.

Key Insights from Present and Future Value Calculations

1. **Time Value of Money**: Present and future value calculations allow individuals and businesses to make informed decisions regarding investments, savings, and loans by recognizing the changing value of money over time.
2. **Investment Growth**: The Future Value formula shows how compound interest can significantly grow investments over time, with higher interest rates and more frequent compounding leading to greater growth.
3. **Discounting Future Cash Flows**: Present Value helps assess the worth of future sums today, which is particularly useful for determining whether a future investment or loan is worth the initial cost.

Task 3

ONLINE DISCUSSION

Use chats, forums, journals, wikis, blogs, question/answer, message my teacher to engage others.

Instructions

a) Carefully solve each of the **six (6)** practice questions provided. Apply the relevant formulas for Present Value (PV) and Future Value (FV) calculations, ensuring you use the correct interest rates and compounding methods as specified in each question.
b) Once you have completed your calculations, collaborate with your classmates to compare your answers. Discuss any differences in your results and the methods you used to arrive at your answers. This peer discussion will help you understand different approaches and reinforce your learning.
c) Post your solutions and explanations in the designated section on the Learning Management System (LMS). Include all necessary calculations and steps you followed to ensure clarity and accuracy in your answers

Practice Questions

1. You invest $2,000 in a savings account that pays 4% annual simple interest. What will be the value of the investment after 5 years?

2. Suppose you invest $3,000 at an annual interest rate of 6%, compounded monthly, for 4 years. Calculate the Future Value of the investment.

3. You wish to have $10,000 saved up in 3 years. If your investment account offers an annual interest rate of 5%, compounded annually, what is the Present Value of the amount you need to invest today?

4. You plan to receive $500 every year for the next 10 years from an investment. The interest rate is 4% annually. What is the Present Value of this annuity?

5. Investment A offers 6% compounded annually, and Investment B offers 5.5% compounded quarterly. If you invest $5,000 in each for 3 years, which investment will yield a higher Future Value, and by how much?
6. A company promises to pay you $1,000 annually forever, starting one year from today. If the discount rate is 5%, what is the Present Value of this perpetuity?

Answers:
1. $2,400
2. $3,837.54
3. $8,638.38
4. $4,051.30
5. Investment A yields $5,955.08, and Investment B yields $5,877.80. Therefore, Investment A yields $77.28 more.
6. Present Value of the Perpetuity = $20,000

Activity 4
Instructions
You are required to watch the on **Loan amortization** video carefully. Pay attention to the steps involved in using the amortization formula. Take short notes on the key points, formulas, and examples provided in the video. Practice using the amortization formula with the example provided in the video or by creating your own scenarios.
Test your understanding by handling quiz/questions labeled **task 4.**

VIDEO 2: Loan amortization

Please click on the link below:
https://youtu.be/lkNJvsy0qU8?si=rpIf0Z1BAfkQp0TU

Task 4

QUIZ/ QUESTIONS

1. What is the correct formula for calculating the monthly payment on a loan?

A.

$$M = P \times \left[\frac{P.r(1+r)^n}{(1+r)^n - 1}\right]$$

B.

$$M = P \times \left[\frac{P.r}{(1+r)}\right]$$

C.

$$M = P \times \left[\frac{P.r.n}{1-(1+r)^{-n}}\right]$$

D.

$$M = \left[\frac{P}{(n.r)}\right]$$

Answer: A. $P \times \left[\frac{P.r(1+r)^n}{(1+r)^n-1}\right]$

2. What does the variable 'r' represent in the amortization formula?

A. Annual interest rate

B. Monthly interest rate

C. Number of payments

D. Principal loan amount

Answer: B. Monthly interest rate

3. How is the annual interest rate used in the amortization formula?

A. It is used directly without modification.

B. It is divided by 12 to get the monthly rate.

C. It is squared to get the monthly rate.

D. It is multiplied by 12 to get the monthly rate.

Answer: B. It is divided by 12 to get the monthly rate.

4. What factors affect the total interest cost of a loan?

A. Principal amount, interest rate, loan term

B. Only the interest rate

C. Only the loan term

D. Only the frequency of payments

Answer: A. Principal amount, interest rate, loan term

5. Which of the following is used to create a loan amortization schedule?

A. Monthly payment calculation only

B. Payment periods, principal paid, interest paid, and remaining balance

C. Only the total interest cost

D. Loan amount and interest rate only

Answer: B. Payment periods, principal paid, interest paid, and remaining balance

6. The amortization formula calculates the monthly payment based on the total interest and principal amount only.

   Answer: False

7. A longer loan term generally results in higher total interest costs.

   Answer: True

8. A loan amortization schedule includes details such as payment periods, interest paid, and remaining balance.

   Answer: True

9. Match the term to its correct description:
   1. Amortization Formula
   2. Monthly Payment (M)
   3. Principal (P)
   4. Monthly Interest Rate (r)
   5. Number of Payments (n)

   Descriptions:

   A. The amount of each loan payment made monthly

   B. The initial loan amount borrowed

   C. The formula used to calculate the regular payment amount for a loan

D. The interest rate per period, often found by dividing the annual rate by 12

E. The total number of payments to be made over the term of the loan

Answers:

1. C. The formula used to calculate the regular payment amount for a loan

2. A. The amount of each loan payment made monthly

3. B. The initial loan amount borrowed

4. D. The interest rate per period, often found by dividing the annual rate by 12

5. E. The total number of payments to be made over the term of the loan

Task 5

END OF MODULE ASSESSMENT

Question

1. Investment X offers 7% compounded annually, while Investment Y offers 6.5% compounded quarterly. If you invest $2,000 in each for 4 years, which investment will yield a higher future value, and by how much?
   Answer: Investment X yields $34.85 more
2. You will receive $800 annually for 8 years. If the annual interest rate is 6%, what is the present value of this annuity?
   Answer: Present Value of Annuity: $4,022.08
3. You want to have $4,000 in 5 years. If the annual interest rate is 7%, compounded annually, what is the present value of this amount?
   Answer: Present Value: $2,841.62

4. You invest $1,200 at an annual interest rate of 5%, compounded semi-annually, for 3 years. What is the future value of the investment?

   Answer: Future Value: $1,391.12

5. You borrow $250,000 for a 30-year mortgage at a 5.5% annual interest rate. What will be your monthly payments?

   Answer: Approximately $1,432.99

6. You are considering two investment options:

- **Option A:** A lump sum investment of $15,000 today, promising a return of $20,000 in 7 years.
- **Option B:** A series of annual investments of $2,500 for 7 years, promising a total return of $18,000.

Which option has a higher net present value (NPV) at a discount rate of 6%?

Answers: Option A: NPV ≈ $1,469.22

Option B: NPV = ≈ $15,613.88

Since Option A has a higher NPV, it is the better investment choice.

REFERENCE LIST AND FURTHER READING

1. Brealey, R. A., Myers, S. C., & Allen, F. (2014). *Principles of corporate finance*. McGraw-hill.
2. Clendenen, G., & Salzman, S. A. (2018). *Business mathematics*. Pearson.
3. Elroi Academy. (2012, October 27). *Example of Simple and Compound interest*. YouTube. https://www.youtube.com/watch?v=E7ZqkxlS2qM
4. Focardi, S. M., Fabozzi, F. J., & Bali, T. G. (2013). *Mathematical Methods for Finance: Tools for Asset and Risk Management*. John Wiley & Sons.
5. Melicher, R. W., & Norton, E. A. (2013). *Introduction to finance: markets, investments, and financial management*. John Wiley & Sons.
6. Steland, A. (2012). *Financial statistics and mathematical finance: methods, models and applications*: John Wiley & Sons.
7. The Organic Chemistry Tutor. (2022, August 16). *Amortization Loan Formula*. YouTube. https://www.youtube.com/watch?v=IkNJvsy0qU8

TAKE HOME MESSAGE

Take a moment to reflect on how the principles we've explored will impact your approach to financial decision-making.
Remember, mastering these concepts equips you with the tools to evaluate investments, manage finances, and optimize financial strategies in real-world scenarios. Carry this knowledge forward and leverage it to make sound financial decisions in your future endeavors.

WHAT'S NEXT?

- Module 6 Review Assignment
- Module 7: Calculus for Business.

Congratulations on completing **Mathematics of Finance** module!

As we move forward, we'll be delving into the fascinating world of calculus, which is essential for understanding and analyzing business problems.

Get ready to:

- Discover the power of calculus in solving complex business problems.
- Apply calculus concepts to real-world scenarios.
- Develop your analytical and problem-solving skills.

Let's embark on this exciting journey together!

MODULE 7: CALCULUS FOR BUSINESS

INSTRUCTIONAL HOURS: 3

MODULE OVERVIEW

In this module, you will explore the fundamental concepts of calculus as they apply to business scenarios. We will begin with the basics of limits and continuity, which are essential for understanding how functions behave and change. You will then cover the core concepts of differentiation and integration, including differentiation rules and procedures, and how to apply derivatives to analyze business situations such as consumer and producer surplus, marginal analysis, and optimization. The module will also address integration rules and procedures, focusing on their practical applications in calculating total cost and revenue. By the end of this module, you will develop the skills to use calculus to solve complex business problems, optimize outcomes, and make informed decisions based on mathematical analysis.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Explain the fundamental concepts of calculus and their relevance to business applications.
2. Demonstrate proficiency in applying differentiation rules and integration procedures to solve mathematical problems and analyze business functions.
3. Utilize calculus techniques to solve business-related problems.
4. Evaluate the role of calculus in business decision-making and its importance in optimizing performance and making data-driven financial decisions.

LEARNING ACTIVITIES/TASK LIST

1. Review power presentation PDF notes
2. Participate in an online discussion
3. Complete participation assignment: Quiz 1
4. Reading material 1: Book chapter
5. Quiz/Questions
6. Reading material 2: Notes
7. Mastery exercise 1
8. Reading material 3: Book chapter
9. Online group discussion
10. Watch video
11. Masterly exercise 2
12. End of module assessment

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning
READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment…)

Activity 1
Review the Notes and upon completion; undertake the quiz or questions **(Task 1)**

Introduction to Calculus

What is Calculus?

Calculus is a branch of mathematics that deals with change and motion. It provides tools to analyze the behavior of functions and models dynamic systems in a wide variety of fields.

In simple terms, calculus helps us study how things change over time, which is essential in many areas of business, such as predicting profits, calculating costs, or optimizing operations.

There are two primary areas of calculus:

1. Differential Calculus: This focuses on the concept of the *derivative*, which measures the rate at which one quantity changes concerning another. It's useful in determining slopes of curves, rates of change (e.g., how fast sales increase), and optimization problems (e.g., maximizing profit or minimizing cost).
2. Integral Calculus: This focuses on the concept of the *integral*, which measures the total accumulation of quantities, such as the total cost or total revenue over a period. It's often used to compute areas under curves, which can represent things like total revenue or total costs accumulated over time.

Illustration of Calculus in Business

Calculus for Business is a specialized field that applies calculus concepts and techniques to solve problems in economics and business.

- Example 1: Marginal Cost (Differentiation).

  Imagine a company producing widgets. The cost of producing these widgets doesn't increase by the same amount with each additional unit produced.
  The *marginal cost* (the cost to produce one more unit) can change, and we can use calculus to determine how these costs vary as production increases. By finding the derivative of the cost function, the company can identify at what point increasing production will become too costly and how to adjust production levels to optimize profits.
- Example 2: Total Revenue (Integration)

A company wants to know its total revenue from selling products over time. The function that describes revenue as it fluctuates can be integrated to determine total revenue. This is where integral calculus comes in, allowing the business to accumulate values over a specific period, aiding in long-term financial planning.

Fundamental Concepts of Calculus

1. Limits

A limit is a foundational concept in calculus, describing the behavior of a function as its input approaches a certain value. In business, limits can be used to analyze scenarios like:

- How production costs behave as output levels approach capacity limits.
- How customer satisfaction changes as the number of customers approaches a certain threshold.

Example: A company might want to analyze how production costs approach a certain value as output increases indefinitely. Understanding this limit helps in long-term cost planning.

2. Derivatives (Differentiation)

A derivative measures the *rate of change* of a function. In business, it allows us to understand how sensitive one variable is to changes in another. For instance, we can calculate:

- The rate at which profits change as the number of units sold increases.
- How the cost of materials changes when the volume purchased increases.

Applications in Business:

- **Marginal Analysis**: A business uses derivatives to determine marginal cost and marginal revenue, helping managers decide whether to increase or decrease production.
- **Optimization**: Businesses often want to maximize profit or minimize cost. Using derivatives, they can find the exact point (called the "critical point") where profit is maximized or cost is minimized.

Example:

If a company's profit function is given by $P(x) = -2x^2 + 30x - 100$, where x is the number of units sold, the derivative $P'(x)$ helps determine at what level of sales the company maximizes its profit.

3. Integrals (Integration)

An integral is used to calculate the *total accumulation* of a quantity over time or another dimension. This is especially useful in business for:

- Finding the total revenue or cost over a given period.
- Calculating the total value of inventory, accumulated depreciation, or total cost of a project over time.

Applications in Business:

- **Accumulated Revenue**: By integrating a revenue function, businesses can calculate total revenue over a specific period, helping them forecast earnings.
- **Cost Calculations**: Businesses use integration to determine the total cost of production, particularly when costs vary with output.

Example:
If the revenue function $R(x) = 10x^2$, integrating this function over a certain range will give the total revenue generated over that range of output. This allows businesses to determine cumulative earnings, which is crucial for financial analysis and decision-making.

4. The Fundamental Theorem of Calculus

This theorem bridges differentiation and integration, showing that they are inverse operations. In business, it connects the rate of change of a function (like how fast costs are increasing) with the total accumulation of that function (like total costs).

Business Applications of Calculus

1. **Optimization of Profit and Cost**
    - Businesses aim to maximize profit and minimize cost. Using derivatives, we can determine the production level at which profit is maximized or the input level at which cost is minimized.

Example: A company with a profit function can use calculus to find the level of sales that leads to the maximum profit by setting the derivative of the profit function equal to zero.

2. **Marginal Analysis**
    - Marginal analysis is widely used in economics and business to make decisions on whether to increase or decrease production. The *marginal cost* and *marginal revenue* derived from calculus help businesses decide the most efficient output level.

Example: If a company knows the cost of producing an additional unit of product, it can compare that to the additional revenue generated to decide whether to increase or decrease production.

3. **Consumer and Producer Surplus**
    - Calculus is used to find consumer and producer surplus in economics, which measure the benefits buyers and sellers receive from transactions in a market.

Example: Integrating demand and supply curves allows businesses and policymakers to assess how much value is created or lost due to market conditions.

4. **Inventory and Depreciation**
    - Integration can be applied to track inventory over time or calculate the total depreciation of assets. Businesses can model the gradual loss in value of assets using calculus.

From the foregoing discussion, you will realize that calculus plays a crucial role in business, helping decision-makers model, analyze, and optimize various processes. From calculating marginal costs and revenues to determining the total accumulated value of revenue or cost over time, calculus provides essential tools for improving profitability, efficiency, and long-term planning.

In this module, you will therefore learn these core calculus concepts and see firsthand how they apply to real-world business scenarios, enabling you to make more data-driven and strategic decisions.

Task 1

ONLINE DISCUSSION

Use chats, forums, journals, wikis, blogs, question/answer, message my teacher to engage others.

Calculus is often viewed as a purely mathematical subject, but its real value lies in its applications to real-world business decisions. Reflect on a specific business scenario where you think calculus (either differentiation or integration) could help a company improve its operations, increase profits, or optimize costs.
How do you think the tools of calculus, like marginal analysis or optimization, could impact business strategy and decision-making in that scenario? Share your thoughts, and feel free to provide examples from industries like manufacturing, finance, or marketing.

Activity 2

READING MATERIAL 1: BOOK CHAPTER

You are required to read Section 1.5 and 1.6 (pages 63-83) on **Limits and Continuity** in the book:
Laurence, D., Hoffmann, B., Gerald, L., & Miners, D. (2013). *Applied Calculus For Business, Economics, and the Social and Life Sciences*. McGraw-Hill Ryerson.
After reading, attempt the quiz/questions labeled **Task 2** and discuss your answers with your peers in the forum.

Task 2

QUIZ/ QUESTIONS

1. Show that the rational function $f(x) = \frac{1+x}{x-2}$ *is continous at* $x = 3$
2. Discuss the continuity of each of the following functions:
   a. $f(x) = \frac{1}{x}$
   b. $g(x) = \frac{x^2-1}{x+1}$
   c. $h(x) = \begin{cases} x+1 & if\ x < 1 \\ 2-x & if\ x \geq 1 \end{cases}$
3. For what value of the constant A is the following function continuous for all real x

$$f(x) = \begin{cases} Ax+5 & if\ x < 1 \\ x^2-3x+4 & if\ x \geq 1 \end{cases}$$

Answer: f is continuous for all x only when A=- 3

Activity 3

READING MATERIAL 1: NOTES

Read the power point notes on differential calculus and attempt masterly exercise labeled **Task 3**

This chapter introduces some of the basic techniques of calculus and their application to economic problems. We shall be concerned here with what is known as the "differential calculus".

The differential calculus

Differentiation is a method used to find the slope of a function at any point. Although this is a useful tool in itself, it also forms the basis for some very powerful techniques for solving optimization problems, which are explained in this and the following chapters.

Recall measuring change in the case of a linear function:

y = a + bx

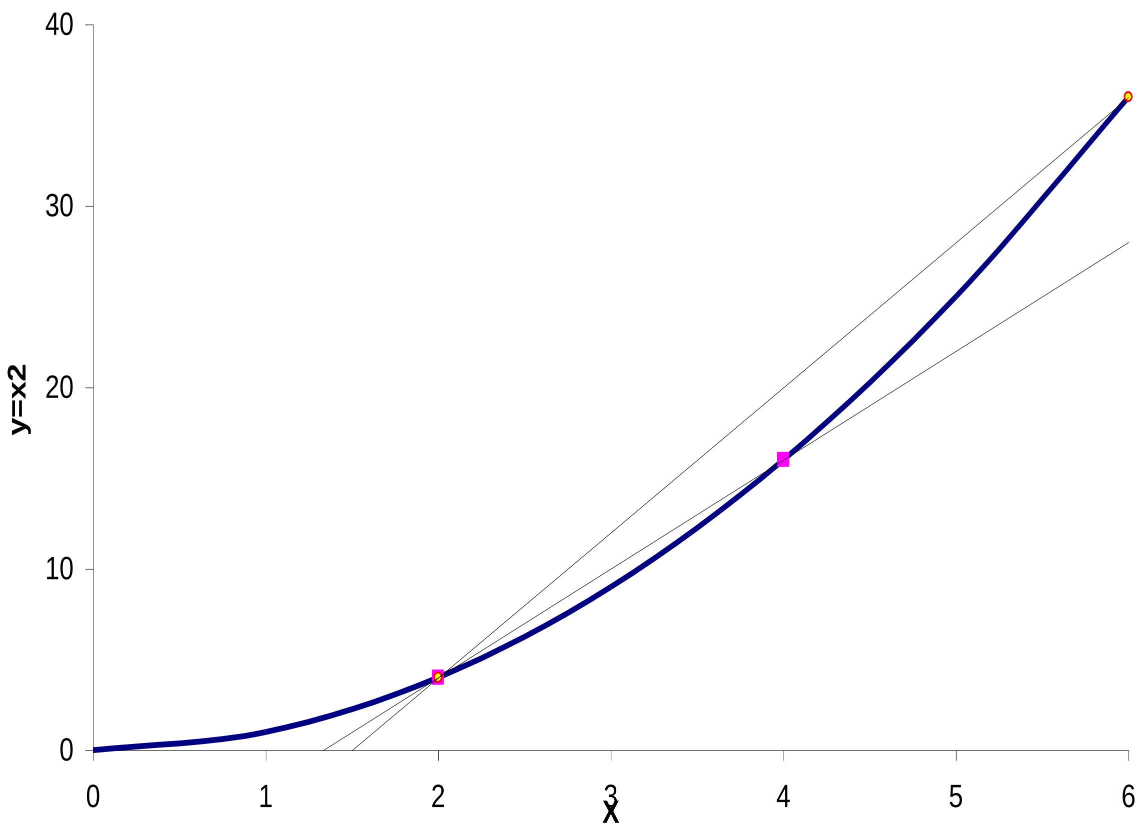
a = intercept

b = slope i.e. the impact of a unit change in x on the level of y

b = $\dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Constant along a straight line; y changes at a constant rate in response to changes in x.

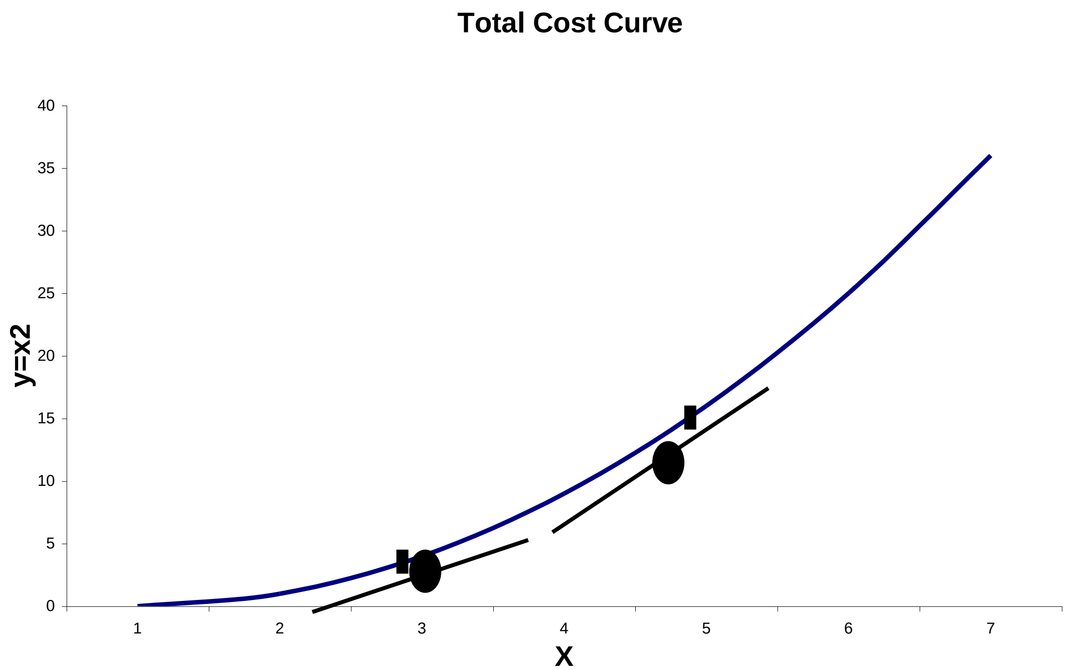
If the function is non-linear: e.g. if y = x^2



$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ gives slope of the *line* connecting 2 points (x_1, y_1) and (x_2,y_2) on a curve

- (2,4) to (4,16): slope = $^{(16-4)}/_{(4-2)}$ = 6
- (2,4) to (6,36): slope = $^{(36-4)}/_{(6-2)}$ = 8

The slope of a curve is equal to the slope of the line (or tangent) that touches the curve at that point; which is different for different values of x



$y = x^2$

$y+\Delta y = (x+\Delta x)^2$

$y+\Delta y = x^2+2x.\Delta x+\Delta x^2$

$\Delta y = x^2+2x.\Delta x+\Delta x^2 - y$

Since $y = x^2$

$\Delta y = 2x.\Delta x+\Delta x^2$

$\frac{\Delta y}{\Delta x} = 2x+\Delta x$

The slope therefore depends on x and Δx

Differentiation: finds the derived function by letting change in x become arbitrarily small, i.e. letting $\Delta x \to 0$

$\frac{\Delta y}{\Delta x}$ = 2x in the limit, as $\Delta x \to 0$

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = 2x$$

Differentiation from the first principle

Differentiate the function *y* = 6x + 2x2 from first principles.

Solution

Assume that x is increased by the small amount Δx (Δ is the Greek letter 'delta' which usually signifies a change in a variable). This will produce a small change Δy in y.

Given the original function

$$y = 6x + 2x2 \qquad (1)$$

The new value of *y* (i.e. *y* + Δ*y*) can be found by substituting the new value of x i.e. $x + \Delta x$) into the function.

Thus

$$y + \Delta y = 6(x + \Delta x) + 2(x + \Delta x)^2$$

$$y + \Delta y = 6x + 6\Delta x + 2x^2 + 4x\Delta x + 2(\Delta x)^2$$

Subtracting (1) $\quad \underline{y \qquad = 6x \qquad + 2x^2}$

gives $\quad \Delta y = 6\Delta x + 4x\Delta x + 2(\Delta x)^2$

Dividing through by Δx,

$$\frac{\Delta y}{\Delta x} = 6 + 4x + 2\Delta x$$

If Δx becomes infinitely small, then the last term disappears and

$$\frac{\Delta y}{\Delta x} = 6 + 4x \qquad (2)$$

By definition, $\mathrm{d}y/\mathrm{d}x$ is the effect of an infinitely small change in x on y. Thus, from (2),

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6 + 4x$$

This is the same result for dy/dx that would be obtained using the basic rules for differentiation explained in the next section.

From the foregoing discussions, it is clear that the basic technique of differentiation is quite straightforward and easy to apply.
Consider the simple function that has only one term

$$y = 6x^2$$

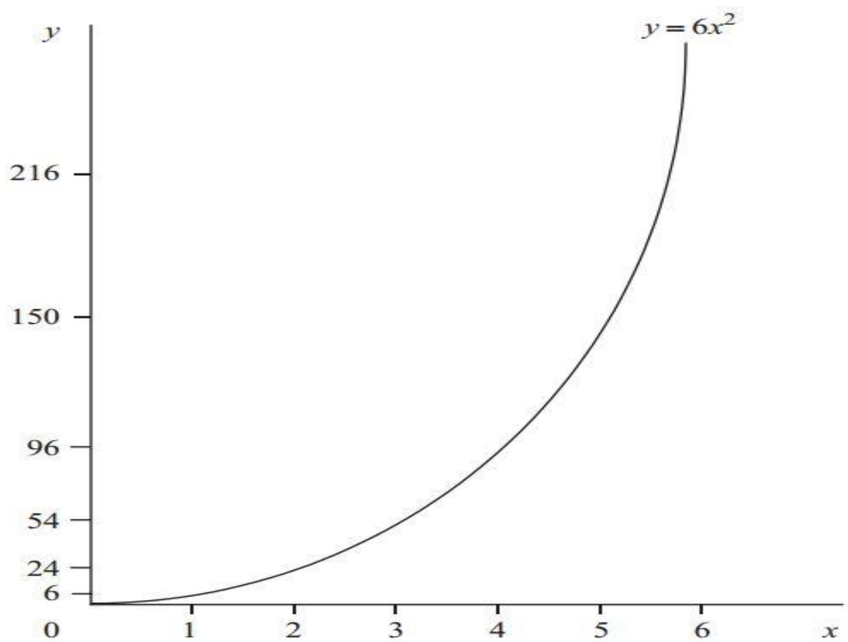
To derive an expression for the slope of this function for any value of x the basic rules of differentiation require you to:

i. Multiply the whole term by the value of the power of x, and
ii. Deduct 1 from the power of x.

In this example there is a term in x^2 and so the power of x is reduced from 2 to 1. Using the above rule the expression for the slope of this function therefore becomes

$2 \times 6x^{2-1} = 12x$

This is known as the derivative of y with respect to x, and is usually written as dy/dx, which is read as 'dy by dx'.



Rules for Differentiation

1. The Constant Rule

If y = c where c is a constant,

$$\frac{dy}{dx} = 0$$

e.g. y = 10 then $\frac{dy}{dx} = 0$

2. The Linear Function Rule

If y = a + bx

$$\frac{dy}{dx} = b$$

e.g. y = 10 + 6x then $\frac{dy}{dx} = 6$

3. The Power Function Rule

If $y = ax^n$, a & n are constants

$$\frac{dy}{dx} = n.a.x^{n-1}$$

Examples

i) y = 4x => $\frac{dy}{dx} = 4x^0 = 4$

ii) $y = 4x^2$ => $\frac{dy}{dx} = 8x$

iii) $y = 4x^3$ => $\frac{dy}{dx} = 12x^2$

iv) $y = 4x^{-2}$ => $\frac{dy}{dx} = -8x^{-3}$

4. The Sum-Difference Rule

If y = f(x) ± g(x)

$$\frac{dy}{dx} = \frac{d[f(x)]}{dx} \pm \frac{d[g(x)]}{dx}$$

If y is the sum/difference of two or more functions of x: differentiate the 2 (or more) terms separately, then add/subtract

Examples

(i) $y = 2x^2 + 3x$ then

$$\frac{dy}{dx} = 4x + 3$$

(ii) $y = 4x^2 - x^3 - 4x$ then $\frac{dy}{dx} = 8x - 3x^2 - 4$

(iii) y = 5x + 4 then $\frac{dy}{dx} = 5$

5. The Product Rule

If y = *u.v* where *u* and *v* are functions of x

Then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

Examples

i) y = (x+2)(ax²+bx)

$$\frac{dy}{dx} = (x+2)(2ax+b) + (ax^2+bx)$$

ii) y = (4x³-3x+2)(2x²+4x)

$$\frac{dy}{dx} = (4x^3-3x+2)(4x+4) + (2x^2+4x)(12x^2-3)$$

6. The Quotient Rule

If y = *u/v* where *u* and *v* are differentiable functions of x

Then $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Examples

i) y = (x+2)/(x+4)

$$\frac{dy}{dx} = \frac{(x+4)-(x+2)}{(x+4)^2} = \frac{2}{(x+4)^2}$$

ii) y = (3x+2)/(x²+4)

$$\frac{dy}{dx} = \frac{(x^2+4)(3)-(3x+2)(2x)}{(x^2+4)^2}$$

$$\frac{dy}{dx} = \frac{-3x^2-4x+12}{(x^2+4)^2}$$

7. The Chain Rule

If y is a function of v, and v is a function of x, then y is a function of x and

$$\frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx}$$

Illustrations

i) $y = (ax^2 + bx)^{½}$

let $v = (ax^2 + bx)$, so $y = v^{½}$

$$\frac{dy}{dx} = \frac{1}{2}\left(ax^2 + bx\right)^{-\frac{1}{2}} . (2ax + b)$$

ii) $y = (4x^3 + 3x - 7)^4$

let $v = (4x^3 + 3x - 7)$, so $y = v^4$

$$\frac{dy}{dx} = 4\left(4x^3 + 3x - 7\right)^3 . \left(12x^2 + 3\right)$$

8. The Inverse Function Rule

If x = f(y) then
$$\frac{dy}{dx} = \frac{1}{dx/dy}$$

The derivative of the inverse of the function x = f(y), is the inverse of the derivative of the function

Examples

(i) $x = 3y^2$ then

$$\frac{dx}{dy} = 6y \quad \text{so} \quad \frac{dy}{dx} = \frac{1}{6y}$$

(ii) $y = 4x^3$ then

$$\frac{dy}{dx} = 12x^2 \quad \text{so} \quad \frac{dx}{dy} = \frac{1}{12x^2}$$

Applications of the Basic Rules of differentiation

Calculating Marginal Functions

Example 1

$$MR = \frac{d(TR)}{dQ} \qquad MC = \frac{d(TC)}{dQ}$$

A firm faces the demand curve P=17-3Q

(i) Find an expression for TR in terms of Q

(ii) Find an expression for MR in terms of Q

Solution:

$TR = P.Q = 17Q - 3Q^2$

$$MR = \frac{d(TR)}{dQ} = 17 - 6Q$$

Example 2:
If a firm's Total Cost Curve is:
$TC = Q^3 - 4Q^2 + 12Q$
Required

i) Find an expression for AC in terms of Q
ii) Find an expression for MC in terms of Q
iii) When does AC=MC?
iv) When does the slope of AC=0?
v) Plot MC and AC curves and comment on the economic significance of their relationship
vi) Suppose now $TC=Q^3- 4Q^2+12Q +10$. Draw new curves and comment….

1) Find the Average Cost

AC = $TC / Q = Q^2 - 4Q + 12$

2) Find the Marginal Cost

$$\frac{d(TC)}{dQ} = 3Q^2 - 8Q + 12$$

3) When does AC = MC?

$Q^2 - 4Q + 12 = 3Q^2 - 8Q + 12$
$\Rightarrow 2Q^2 - 4Q = 0$
$\Rightarrow 2Q = 4$
$\Rightarrow Q = 2$
Thus, AC = MC curves when Q = 2

4) When does the slope of AC = 0?

Differentiate $AC = Q^2 - 4Q + 12$ to find slope……

$$\frac{d(AC)}{dQ} = 2Q - 4$$

Then set it equal to 0
$2Q - 4 = 0$
$\Rightarrow Q = 2$ when slope AC = 0

(v) Economic Significance?
MC curve cuts the AC curve at its minimum point…….(draw both curves)
MC cuts AC curve at minimum point…

(vi) What happens if we introduce Fixed costs to the TC function?
$TC=Q^3- 4Q^2+12Q +10$
$\Rightarrow$ No impact on the MC function,

$\Rightarrow$ Shift up AC function by FC/q

Example 3: Elasticity

Price Elasticity of Demand:

$$e_d = \frac{proportional\ change\ in\ demand}{proportional\ change\ in\ price}$$

$$= \frac{\Delta Q}{Q} / \frac{\Delta P}{P} = \frac{\Delta Q}{\Delta P} . \frac{P}{Q}$$

To calculate the point elasticity of demand then,

$$e_d = \frac{dQ}{dP} . \frac{P}{Q}$$

e.g. Find e_d of the function $Q = aP^{-b}$

$$e_d = -baP^{-b-1} . \frac{P}{aP^{-b}}$$

$$= \frac{-baP^{-b}}{P} . \frac{P}{aP^{-b}} = -b$$

- Inelastic demand: if $|e_d| < 1$
- Unit elastic demand: if $|e_d| = 1$
- Elastic demand: if $|e_d| > 1$

Profit maximization

We are now ready to see how calculus can help a firm to maximize profits, as the following examples illustrate. At this stage we shall just use the MC = MR rule for profit maximization. The second condition (MC cuts MR from below) will be dealt with in subsequent chapters.

Example

A monopoly faces the demand schedule $p = 460 - 2q$ and the cost schedule $TC = 20 + 0.5q^2$

How much should it sell to maximize profit and what will this maximum profit be? (All costs and prices are in £.)

Solution

To find the output where MC = MR we first need to derive the MC and MR functions.

$$\text{Given} \quad TC = 20 + 0.5q^2$$

$$\text{then} \quad MC = \frac{dTC}{dq} = q \qquad (1)$$

$$\text{As} \quad TR = pq = (460 - 2q)q = 460q - 2q^2$$

$$\text{then} \quad MR = \frac{dTR}{dq} = 460 - 4q \qquad (2)$$

To maximize profit MR = MC. Therefore, equating (1) and (2),

$$\begin{aligned} 460 - 4q &= q \\ 460 &= 5q \\ 92 &= q \end{aligned}$$

The actual maximum profit when the output is 92 will be

$$\begin{aligned} TR - TC &= (460q - 2q^2) - (20 + 0.5q^2) \\ &= 460q - 2q^2 - 20 - 0.5q^2 \\ &= 460q - 2.5q^2 - 20 \\ &= 460(92) - 2.5(8{,}464) - 20 \\ &= 42{,}320 - 21{,}160 - 20 = £21{,}140 \end{aligned}$$

Partial Differentiation

In the previous discussion we learned how to use the differential of a function to approximate the change in the function resulting from a small change in its independent variable. Now we extend this concept of differential to a function of two or more independent variables.

Let *Z* be a function of two variables *x* and *y* and the function is smooth and differentiable everywhere.

$$Z = f(x, y)$$

If Δx and Δy denote a small change in x and *y* respectively then the change in Z is given as

$$\Delta Z \approx \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

The partial derivative $\frac{\partial z}{\partial x}$ measures the rate of change of *Z* with respect to an infinitesimal change in x. Similarly $\frac{\partial z}{\partial y}$ measures the rate of change of *Z* with respect to an infinitesimal change in *y*. Hence the change in *Z* due to a change in *x* may be represented by the expression

$\frac{\partial z}{\partial x}dx$ and $\frac{\partial z}{\partial y}dy$ for that of a change in *y*.

Therefore the total change in Z will be equal to the sum of these two partial changes.

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

The process of finding such a total differential is called *total differentiation*.

The more general case of a function of n independent variables can be exemplified by

$Z = f(x_1, x_2, x_3, ---, x_n)$.

The total differential of this function can be written as

$$dz = \frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + ----+ \frac{\partial f}{\partial x_n}dx_n$$

$$or \quad dz = f_1 dx_1 + f_2 dx_2 + ---+ f_n dx_n$$

This shows *dz* or the total change in z is the sum of the n partial changes resulting from all possible sources of change.

Examples 1: Find the total differential of the function $Z = 3x^2 + xy - 2y^3$

Solution: Applying the approximation formula

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy \quad \text{we get}$$

$$dz = (6x + y)dx + (x - 6y^2)dy$$

Example 2: Find the total differential of the function

$$y = \frac{x_1}{x_1 + x_2}$$

Solution: Applying the approximation formula

$$dy = \frac{\partial y}{\partial x_1}dx_1 + \frac{\partial y}{\partial x_2}dx_2, \text{ we obtain}$$

$$dy = \left[\frac{1(x_1 + x_2) - x_1(1)}{(x_1 + x_2)^2}\right]dx_1 + \left[\frac{0(x_1 + x_2) - x_1(1)}{(x_1 + x_2)^2}\right]dx_2$$

$$dy = \left[\frac{x_2}{(x_1 + x_2)^2}\right]dx_1 + \left[\frac{-x_1}{(x_1 + x_2)^2}\right]dx_2$$

$$dy = \left[\frac{x_2}{(x_1 + x_2)^2}\right]dx_1 - \left[\frac{x_1}{(x_1 + x_2)^2}\right]dx_2$$

Rules of partial Differentials

We found the total differential *dy*, given a function $y = f(x_1, x_2)$ by finding the partial derivatives f_1 and f_2 and substitute these in to the equation

$dy = f_1 dx_1 + f_2 dx_2$

But sometimes it may be more convenient to apply certain rules of differentials. Let *c* be a constant and *u* and *v* be two functions of the variables x_1 and x_2.

1. $dc = 0$
2. $d(cu^n) = cnu^{n-1}du$
3. $d(u \pm v) = du \pm dv$
4. $d(uv) = vdu + udv$
5. $d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}$

Example1: Find the total differential of the function $y = 3x_1^2 + x_1x_2^2$ by applying the above rules of differentials

Solution:

$dy = d(3x_1^2) + d(x_1x_2^2)$ (by rule 3).

$= 6x_1dx_1 + x_2^2\,dx_1 + x_1d(x_2^2)$ (by rule 2 and rule 4)

$= 6x_1dx_1 + x_2^2dx_1 + 2x_1x_2dx_2$ (by rule 2)

$= (6x_1 + x_2^2)dx_1 + 2x_1x_2dx_2$

The above total differential can be found directly by finding the partial derivatives

$dy = f_1dx_1 + f_2dx_2$

$\sin ce$

$$f_1 = \frac{\partial f}{\partial x_1} = 6x_1 + x_2{}^2 and f_2 = \frac{\partial f}{\partial x_2} = 2x_1x_2$$

$dy = (6x_1 + x_2^2)dx_1 + 2x_1x_2dx_2$

Therefore students may use a method, which they think is simpler to find the total differential *dy*.

Example 2. Find the total differential *dy* for the function

Solution: $Z = f(x, y) = 3x^3 - x^2y + y + 17$

$dZ = d(3x^3) - d(x^2y) + dy + d(17)$ (by rule 3)

$= 9x^2\,dx - x^2dy - yd(x^2) + dy$ (by rule 1, 2 and 4)

$= 9x^2dx - x^2dy - y(2x)\,dx + dy$ (by rule 2)

$= 9x^2dx - 2xydx - x^2dy + dy$

$= (9x^2 - 2xy)\,dx - (x^2-1)\,dy$

Task 3

MASTERY EXERCISE 1.1

1. Differentiate the function from first principles.

$$y = 3x^2 + 2x$$

2. Differentiate the following function using the quotient rule

$$y = \frac{x^2 + 1}{x - 2}$$

3. Differentiate the function using the product rule

$$y = (x^2 + 3)(2x + 4)$$

4. Differentiate the function below using the chain rule.

$$y = (3x^2 + 2x + 1)^5$$

5. Find the partial derivatives and the total differential of the function:

$$z = 2x^3 - 4xy^2 + 3y + 5$$

6. The demand function for a product is:

$$Q = 100 - 2P$$

where Q is quantity demanded and P is price.

The total cost function is:

$$TC = 100 + 20Q + Q^2$$

I. Find the marginal revenue (MR) and marginal cost (MC) functions.
II. Determine the profit-maximizing level of output and price.

7. A company's production function is given by:

$$Q = 2L^{0.5}K^{0.5}$$

where Q is output, L is labor, and K is capital.

i. Find the marginal product of labor (MPL) and the marginal product of capital (MPK).
ii. Evaluate MPL and MPK at the point (L, K) = (100, 144).

8. Given the total cost function

$$TC = Q^3 + 4Q^2 + 12$$

Find:

i) The marginal cost (MC).
ii) The average cost (AC).
iii) The quantity Q where AC=MC

Answers

1. $\frac{dy}{dx} = 6x + 2$ (should follow the differentiation from the first principle procedure i.e If x increases by a small amount Δx, the new value of y is y+Δy)
2. $\frac{dy}{dx} = \frac{x^2-4x-1}{(x-2)^2}$
3. $\frac{dy}{dx} = 6x^2 + 8x + 6$
4. $\frac{dy}{dx} = 5(3x^2 + 2x + 1)^4.(6x + 2)$
5. $dz = (6x^2 - 4y^2)\, dx + (-8xy + 3)\, dy$
6. i. MR = dTR/dP = 100 - 4P; MC = dTC/dQ = 20 + 2Q
   ii. P = 20, Q = 60
7. MPL = 1/12; MPK = 5/12
8. i. $MC=3Q^2-8Q+12$ ii. $AC=Q^2-4Q+12$ iii. Q=2 is the quantity where AC=MC

Activity 4

READING MATERIAL 2: BOOK CHAPTER

Please read **Chapter 8: Integral Calculus**, specifically focusing on the Rules of Integration from pages 420-436 in the textbook: Ummer, E. K. (2012). Basic Mathematics for Economics, Business, and Finance. Routledge.

Make concise notes summarizing the key rules and formulas. This will be helpful for future reference. Participate on **Task 4: Discussion forum**

Task 4

ONLINE DISCUSSION

Use chats, forums, journals, wikis, blogs, question/answer, message my teacher to engage others.

After reading and making notes on the **Rules of Integration** from pages 420-436, apply your understanding by working through the following tasks:

1. Individually, review **Application Examples 1-6** in the chapter and analyze how integration is used in each case.
2. In groups, discuss the real-world relevance of each example. How can these integration applications be applied to solving economic, business, or financial problems? Share your insights and explanations with each other.
3. After your group discussions, each member should write a brief summary (1-2 paragraphs) explaining one example and how the integration technique was applied to solve it.

Activity 5

VIDEO 1: Definite Integration

Now that we've explored the concept of integration and indefinite integrals, it's time to delve deeper into a crucial aspect of calculus: definite integrals. These integrals help us calculate the total area, volume, or other quantities under a curve over a specific interval.

To get started with definite integrals, we'll be watching a helpful video that explains the concept in detail.

Please click on the following link to watch the video:

https://youtu.be/d4CdMoucVfs?si=dGH7vcad5NlAd0xW

Before watching the video, consider the following questions:

- How is definite integration different from indefinite integration?
- What are some real-world applications of definite integrals?

After watching the video, be prepared to discuss:

- The key concepts of definite integrals, such as the definite integral symbol, integrand, limits of integration, and the connection to indefinite integrals.
- Examples presented in the video of how definite integrals are used to calculate areas and other quantities.
- Any questions or points that require further clarification on definite integrals.
- Undertake **task 5: Masterly exercise**

Task 5

MASTERY EXERCISE 1.2

a) Evaluate the definite integral

$$\int_0^1 (x^2 + 3x)\, dx$$

b) Evaluate

$$\int_1^2 (x^3 - 4x^4 + 14)^5 (3x^2 - 16x^3)\, dx$$

c) Evaluate the following integral

$$\int_{1}^{3} \frac{2y-4}{y^2-4y}.dy$$

d) Evaluate

$$\int_{2}^{3} x^2(x^3-4)^8.dx$$

Task 6

END OF MODULE ASSESSMENT 1.2

1. Apply the rules of differentiation to find the derivatives of the following functions:

   - $y = 4x^3$
   - $y = \frac{x+2}{x+4}$
   - $y = (3x+2)(x^2+4x)$

Answers:

- $\frac{dy}{dx} = 12x^2$
- $\frac{dy}{dx} = \frac{(x+4)-(x+2)}{(x+4)^2}$
- $\frac{dy}{dx} = (6x+2)(x+2) + (3x+2)(2x+4)$

2. Differentiate the function $y = 6x + 2x^2$ from the first principle

   Answer: Students should apply the definition of the derivative using limits, expanding and simplifying terms to find the derivative;

$$\frac{dy}{dx} = 6 + 4x$$

3. Determine if the function $f(x) = \frac{1+x}{x-2}$ is continuous at x=3. Show your calculations and reasoning.

   Answer: Students should show the steps for evaluating the function at x=3, checking if the limit of f(x)as x approaches 3 from both sides exists and equals f(3).

4. The demand function for a product is:

$$Q = 100 - 2P$$

where Q is quantity demanded and P is price.

The total cost function is:

$$TC = 100 + 20Q + Q^2$$

i. Find the marginal revenue (MR) and marginal cost (MC) functions.
ii. Determine the profit-maximizing level of output and price

Answers: i. MR = dTR/dP = 100 - 4P; MC = dTC/dQ = 20 + 2Q
ii. P = 20, Q = 60

REFERENCE LIST AND FURTHER READING

1. Hoffmann, L., Bradley, G., Sobecki, D., & Price, M. (2012). *EBOOK: Applied Calculus for Business, Economics and the Social and Life Sciences*. McGraw Hill.
2. Harrison, M., & Waldron, P. (2011). *Mathematics for economics and finance*. Routledge.
3. Laurence, D., Hoffmann, B., Gerald, L., & Miners, D. (2013). *Applied Calculus For Business, Economics, and the Social and Life Sciences*. McGraw-Hill Ryerson.
4. The Organic Chemistry Tutor. (2019, December 21). *Calculus - Definite Integrals*. YouTube. https://youtu.be/d4CdMoucVfs?si=dGH7vcad5NlAd0xW
5. Ummer, E. K. (2012). *Basic mathematics for economics, business, and finance*. Routledge (pg 420-436).

TAKE HOME MESSAGE

Mastering calculus isn't just about solving equations; it's about unlocking powerful tools that help you make informed business decisions, optimize resources, and predict future trends.
As you apply these concepts, remember that every derivative and integral can lead to smarter strategies and greater business efficiency.

WHAT'S NEXT?

- Module 7 Review Assignment
- Module 8: Decision Analysis
- Module 8: Decision Analysis
- In our next module, get ready to dive into the dynamic world of financial systems and their role in economic growth. Embrace the learning journey and equip yourself with valuable insights for your future endeavors. Enjoy and engage actively.

MODULE 8: DECISION ANALYSIS

INSTRUCTIONAL HOURS: 4

MODULE OVERVIEW

In this module, you will explore the principles and techniques of decision analysis, which are crucial for making informed decisions in uncertain and complex environments. We will begin by exploring decision-making under uncertainty, where you will learn on strategies and methodologies for evaluating options when outcomes are not guaranteed. You will then study expected value and decision trees, powerful tools for quantifying and visualizing potential outcomes and their probabilities, aiding in rational decision-making. Additionally, we will delve into risk analysis, examining methods to evaluate and manage risks associated with various decisions. By the end of this module, you will be equipped with the analytical skills needed to make strategic decisions, manage risks effectively, and apply decision analysis techniques to real-world scenarios.

LEARNING OUTCOMES

By the end of this module, you will be able to:

1. Explain the key concepts in decision analysis.
2. Calculate and apply expected value and decision tree methods to analyze various decision scenarios.
3. Utilize risk analysis techniques to assess and manage potential risks in decision-making processes, applying these methods to make informed business decisions.
4. Evaluate the role of structured decision analysis in improving decision-making quality, managing uncertainty and mitigating risks in business contexts.

LEARNING ACTIVITIES/TASK LIST

1. Listen to a pre-recorded lecture
2. Watch YouTube video
3. Review power presentation PDF: Lecture notes
4. Complete participation assignment: Quiz 1
5. Watch video
6. Masterly exercise 1.1
7. Reading material 2: Book chapter
8. End of module assessment 1.2

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning
READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment...)

Activity 1
You are required to listen to the prerecorded lecture on the Introduction to Decision Theory
Upon completion; watch the video on Decision-Making under Uncertainty (Task 1) and participate in the subsequent discussion.

Introduction to Decision Theory

In the previous topics, we've primarily explored decision-making scenarios where outcomes are relatively predictable and the consequences of alternative actions can be estimated with some degree of certainty. This enabled the use of mathematical models such as linear programming and others to optimize decisions based on available data.

While these models often provide valuable insights, they rely on the assumption that outcomes can be reasonably forecasted.

However, the real world rarely operates with this level of certainty. In many cases, decision-makers must choose between alternatives in environments filled with uncertainty and risk, where the outcomes are unknown and cannot be easily predicted. This is where decision theory comes into play.

Decision theory focuses on making choices in situations where the future is uncertain. It provides frameworks, tools, and strategies to analyze possible actions and their uncertain consequences, enabling more informed decisions even when complete information is unavailable.

Examples of Decision-Making under Uncertainty:

1. Introducing a New Product: A company planning to launch a new product must decide how much to produce without knowing how customers will react. Should they test the product in a smaller market before committing to full production? How much marketing should be invested upfront to ensure a successful launch?

2. Financial Investments: An investment firm must decide which stocks or market sectors to invest in, despite uncertainty about future economic trends, interest rates, and the performance of different sectors.
3. Bidding on Government Contracts: A contractor bidding for a new project faces uncertainty about the actual costs involved, as well as the competition's bidding strategies. What are the best strategies to win the bid while ensuring profitability?
4. Agriculture Planning: Farmers must decide on the mix of crops and livestock for the upcoming season without knowing what weather conditions will prevail, how market prices will change, or what production costs will be.
5. Oil Drilling Decisions: An oil company considering drilling in a new location faces uncertainty regarding the likelihood of finding oil, the amount available, and the depth required. Should they spend more time investigating the site before committing to drilling?

These examples illustrate how uncertainty can complicate decision-making in various industries.

Decision theory is a field of study that provides a systematic approach to making decisions in situations where the outcomes are uncertain.

This field provides structured approaches, such as decision trees, expected value calculations, and risk analysis, to help navigate these complex decisions.

Key benefits of Studying Decision Theory in Business

By delving into decision theory, you'll gain critical tools and insights that will empower you to navigate complex business environments. Understanding this subject will provide you with a strategic edge, enabling you to make informed, data-driven decisions that drive success.

Here are the key benefits you'll unlock through studying decision theory in business:

1. Improved Decision-Making: Understanding decision theory helps business professionals make more rational, well-thought-out decisions, especially in environments where uncertainty and risk are involved.

2. Risk Management: Businesses often face risky situations, and decision theory equips decision-makers with the tools to assess and mitigate risks effectively, enhancing overall stability and resilience.
3. Optimal Resource Allocation: With decision theory, businesses can allocate resources more efficiently, ensuring the highest return on investment by weighing various possible outcomes and selecting the best course of action.
4. Strategic Advantage: Businesses that apply decision theory can gain a competitive edge by making more informed and proactive decisions, allowing them to adapt to changing market conditions and uncertainty more effectively.
5. Enhanced Problem-Solving: Decision theory encourages structured problem-solving by breaking down complex decisions into manageable steps, which helps in identifying the best options in business situations.
6. Quantitative Decision Support: The use of quantitative tools such as decision trees and expected value calculations in decision theory allows businesses to base decisions on solid data and analysis rather than intuition alone.
7. Better Financial Planning: Decision theory helps companies make better financial decisions by considering multiple future scenarios and determining the optimal financial strategy under uncertainty.

Provoking visual icebreaker

Imagine standing at a crossroads with three different paths ahead. One path leads to the mountains, another to the sea, and the third to a forest. You know little about what lies ahead on each path, but you must choose one.

Now, think: How would you make that decision?

This visual metaphor illustrates the essence of decision theory. When faced with uncertainty, our task is to weigh potential outcomes, use available information, and make informed decisions. In the world of business, we often find ourselves at similar crossroads, and this module will equip you with the tools to navigate those choices successfully.

As we embark on this journey into decision theory, get ready to unlock powerful tools that will enhance your ability to make well-informed, strategic choices in uncertain environments.

Embrace the learning process with an open mind, as the concepts you are about to explore will not only sharpen your decision-making skills but also prepare you for real-world challenges in the business landscape.

Let's dive in and make every decision count!

Task 1

VIDEO 1: Decision-Making Under Uncertainty

Watch the video: https://youtu.be/XYtnnOjLEno?si=VYormpRX0OS-S-pI
After watching:

1. Reflect on real-life business situations where uncertainty plays a crucial role in decision-making (e.g., product launches, investments, bidding for contracts, or agricultural planning).
2. Choose one of the following scenarios (or propose your own) to discuss:

i. A company deciding to launch a new product without knowing how the market will respond.
ii. An investor deciding which stocks to buy in an unpredictable market.
iii. A government contractor unsure of the actual costs for a project they are bidding on.
iv. A farmer deciding what crops to plant, uncertain about future weather and market prices.

3. Post your thoughts on how decision theory and decision-making tools (like decision trees and expected value) can help in such uncertain scenarios.

Discussion Prompts

- What uncertainties exist in your chosen scenario?
- How would you approach this decision using expected value, decision trees, or other tools discussed in class?
- What strategies would help mitigate the risks in this scenario?

Participation Requirement:

- Post your initial response and engage with at least two peers' posts by providing constructive feedback or offering alternative approaches to their scenario.

Activity 2

READING MATERIAL 1: LECTURE NOTES

Review the Notes and upon completion; undertake the quiz or questions (Task 2)

Decision-Making Criteria under Uncertainty: Analyzing Maximax, Maximin, and Minimax Regret Criteria

Decision-making under uncertainty often requires evaluating various approaches to determine the best course of action when future outcomes are not guaranteed.

Among the various strategies, the Maximax Criterion, Maximin Criterion, and Minimax Regret Criterion offer distinct methods for assessing risk and reward.

The Maximin Criterion, or pessimistic approach, focuses on minimizing potential losses by selecting the alternative with the best worst-case outcome. Conversely, the Maximax Criterion, or optimistic approach, seeks to maximize potential gains by choosing the alternative with the highest best-case payoff.

The Minimax Regret Criterion, on the other hand, aims to minimize the maximum possible regret by evaluating the regret associated with each decision alternative.

Understanding these criteria helps decision-makers navigate uncertainty by aligning their choices with their risk tolerance and strategic goals.

1. Maximax Criterion (Optimistic Approach)

Definition: The Maximax Criterion is used by optimistic decision-makers who believe in the best possible outcomes. It focuses on maximizing the maximum possible gain.

Steps to Apply:

i. List the Payoffs: Create a payoff matrix with different decision alternatives and states of nature.
ii. Identify the Best Payoff for Each Alternative: For each decision alternative, identify the best (maximum) payoff possible.
iii. Choose the Alternative with the Highest of the Best Payoffs: Select the decision alternative that has the highest value among the best payoffs.

Illustration:

Using the same payoff matrix:

Project/Condition	Good Economy	Average Economy	Poor Economy
A	$100,000	$60,000	$20,000
B	$80,000	$50,000	$10,000
C	$90,000	$40,000	$30,000

Identify the Best Payoff for Each Project:
Project A: $100,000
Project B: $80,000
Project C: $90,000

Choose the Highest of the Best Payoffs:
The best payoff for Project A is $100,000.
The best payoff for Project B is $80,000.
The best payoff for Project C is $90,000.

Therefore, the best decision according to the Maximax Criterion is Project A, which has the highest maximum payoff.

2. Maximin Criterion (Pessimistic Approach)

Definition: The Maximin Criterion is a decision-making approach used when the decision-maker is pessimistic about future states. It focuses on minimizing the maximum possible loss. This approach is also known as the "pessimistic" approach because it assumes the worst-case scenario.

Steps to Apply:

i. List the Payoffs: Create a payoff matrix with different decision alternatives and states of nature.
ii. Identify the Worst Payoff for Each Alternative: For each decision alternative, identify the worst (minimum) payoff possible.
iii. Choose the Alternative with the Best of the Worst Payoffs: Select the decision alternative that has the highest value among the worst payoffs.

Illustration:

Consider a company deciding on which project to invest in. The projects and their potential payoffs under different economic conditions are:

Project/Condition	Good Economy	Average Economy	Poor Economy
A	$100,000	$60,000	$20,000
B	$80,000	$50,000	$10,000
C	$90,000	$40,000	$30,000

Identify the Worst Payoff for Each Project:
Project A: $20,000
Project B: $10,000
Project C: $30,000

Choose the Best of the Worst Payoffs:
The worst payoff for Project A is $20,000.
The worst payoff for Project B is $10,000.
The worst payoff for Project C is $30,000.

Therefore, the best decision according to the Maximin Criterion is Project C, which has the highest minimum payoff.

3. Minimax Regret Criterion

Definition: The Minimax Regret Criterion is used to minimize the maximum regret that could result from making a decision. Regret is defined as the difference between the payoff of the best decision in hindsight and the payoff of the chosen decision.

Steps to Apply:

i. List the Payoffs: Create a payoff matrix with different decision alternatives and states of nature.
ii. Calculate the Regret Matrix: For each decision alternative and state of nature, calculate the regret by subtracting the payoff of that decision alternative from the best payoff for that state of nature.
iii. Identify the Maximum Regret for Each Alternative: For each decision alternative, identify the maximum regret.
iv. Choose the Alternative with the Minimum of the Maximum Regrets: Select the decision alternative with the lowest maximum regret.

Illustration:

Using the same payoff matrix:

Project/Condition	Good Economy	Average Economy	Poor Economy
A	$100,000	$60,000	$20,000
B	$80,000	$50,000	$10,000
C	$90,000	$40,000	$30,000

Determine the Best Payoff for Each State:
Good Economy: $100,000 (Project A)
Average Economy: $60,000 (Project A)
Poor Economy: $30,000 (Project C)

Calculate the Regret Matrix:

Project/Condition	Good Economy	Average Economy	Poor Economy
A	$0	$0	$10,000
B	$20,000	$10,000	$20,000
C	$10,000	$20,000	$0

Identify the Maximum Regret for Each Project:
Project A: $10,000
Project B: $20,000
Project C: $20,000

Choose the Alternative with the Minimum of the Maximum Regrets:
The maximum regret for Project A is $10,000.
The maximum regret for Project B is $20,000.

The maximum regret for Project C is $20,000.

Therefore, the best decision according to the Minimax Regret Criterion is Project A, which has the lowest maximum regret.

These criteria help decision-makers choose the best alternative based on different attitudes towards risk and uncertainty.

Expected Value in Decision Theory

Definition: In decision theory, the Expected Value (EV) is a key concept used to make decisions under uncertainty by calculating the weighted average of all possible outcomes, considering their probabilities. It provides a measure of the central tendency of the possible outcomes, helping decision-makers evaluate and compare different choices based on their potential benefits and risks.

Formula: The Expected Value (EV) of a decision alternative is given by:

$EV = \sum_{i=1}^{n}[x_i \cdot P(x_i)]$

where:

- x_i = outcome value of the i-th scenario
- $P(x_i)$ = probability of the i-th scenario occurring
- n = number of possible scenarios

Illustration with Decision-Making Example

Scenario: A company is considering launching a new product. The success of the product depends on three possible market conditions: high demand, moderate demand, and low demand. The financial outcomes and probabilities associated with each condition are as follows:

- **High Demand:** Profit of $500,000 with a probability of 0.3
- **Moderate Demand:** Profit of $200,000 with a probability of 0.5
- **Low Demand:** Loss of $100,000 with a probability of 0.2

Objective: Calculate the Expected Value of launching the new product to help the company make an informed decision.

Steps to Calculate Expected Value:

1. **List the Possible Outcomes and Their Probabilities:**
    i. High Demand: Profit = $500,000, Probability = 0.3
    ii. Moderate Demand: Profit = $200,000, Probability = 0.5

iii. Low Demand: Loss = -$100,000, Probability = 0.2

2. **Apply the Expected Value Formula:**

$EV = (500{,}000 \cdot 0.3) + (200{,}000 \cdot 0.5) + (-100{,}000 \cdot 0.2)$

3. **Perform the Calculations:**

$EV = (150{,}000) + (100{,}000) + (-20{,}000)$

$EV = 150{,}000 + 100{,}000 - 20{,}000$

$EV = 230{,}000$

Answer: The Expected Value of launching the new product is $230,000.

Decision-Making Under Uncertainty

In decision theory, Expected Value is used to:

1. **Compare Alternatives:** Decision-makers can compare the expected values of different options to choose the one with the highest EV, assuming all other factors are equal.
2. **Assess Risk and Benefit:** By calculating the EV, decision-makers can evaluate the average potential outcome of a decision, helping to understand the potential benefits and risks.
3. **Guide Strategic Planning:** EV helps in strategic planning by providing a numerical basis for making decisions that align with the organization's goals and risk tolerance.

Applications

1. **Investment Decisions:** Investors use EV to evaluate the potential returns of different investment opportunities, balancing risk and reward.
2. **Business Strategies:** Companies use EV to assess the financial impact of strategic decisions, such as product launches, market expansions, or cost-saving measures.
3. **Policy Making:** Policymakers use EV to evaluate the potential outcomes of different policies or regulations, considering their economic and social impacts.

Limitations

1. **Does Not Account for Risk Tolerance:** EV provides an average outcome but does not consider an individual's or organization's risk tolerance. Two options with the same EV might have different risk profiles.
2. **Assumes Accurate Probabilities:** The accuracy of the EV calculation depends on the correct estimation of probabilities. Incorrect probabilities can lead to misleading results.
3. **Does Not Capture All Aspects:** EV focuses on average outcomes and may not capture other important factors such as qualitative aspects, ethical considerations, or long-term implications.

Expected Value is a fundamental concept in decision theory used to evaluate and compare different decision alternatives under uncertainty. By calculating the weighted average of possible outcomes, it helps decision-makers make informed choices based on potential risks and benefits. However, it is important to consider other factors such as risk tolerance and probability accuracy when using EV in decision-making.

Task 2

QUIZ/ QUESTIONS

1. A company is considering three different projects: Project X, Project Y, and Project Z. The potential profits under different economic conditions are given in the following payoff matrix:

Project/Condition	Boom Economy	Stable Economy	Recession Economy
X	$200,000	$100,000	$20,000
Y	$150,000	$120,000	$30,000
Z	$180,000	$90,000	$50,000

a. Using the Maximin Criterion, which project should the company choose?

b. Using the Maximax Criterion, which project should the company choose?

c. Using the Minimax Regret Criterion, which project should the company choose?

Answers

a. **Maximin Criterion:**

Answer: The best choice using the Maximin Criterion is Project Z.

b. **Maximax Criterion:**

Answer: The best choice using the Maximax Criterion is Project X.

c. **Minimax Regret Criterion:**

Answer: The best choice using the Minimax Regret Criterion is either Project X or Project Z, both with a maximum regret of $30,000.

2. A retailer must decide on an advertising campaign with three options: Campaign A, Campaign B, and Campaign C. The potential revenues under different market conditions are provided in the following matrix:

Campaign/Condition	High Market	Moderate Market	Low Market
A	$300,000	$150,000	$50,000
B	$250,000	$200,000	$40,000
C	$270,000	$180,000	$70,000

a. Using the Maximin Criterion, which campaign should the retailer choose?

b. Using the Maximax Criterion, which campaign should the retailer choose?

c. Using the Minimax Regret Criterion, which campaign should the retailer choose?

Answers

a. **Maximin Criterion:**

Answer: The best choice using the Maximin Criterion is Campaign C.

b. **Maximax Criterion:**

Answer: The best choice using the Maximax Criterion is Campaign A.

c. **Minimax Regret Criterion:**

Answer: The best choice using the Minimax Regret Criterion is Campaign C.

3. A company is considering investing in one of two new projects: Project A or Project B. The potential profits and probabilities for each project are as follows:

- **Project A:**
    - High Profit: $400,000 with a probability of 0.4
    - Medium Profit: $150,000 with a probability of 0.4
    - Low Profit: $50,000 with a probability of 0.2
- **Project B:**

- High Profit: $300,000 with a probability of 0.3
- Medium Profit: $250,000 with a probability of 0.5
- Low Profit: $100,000 with a probability of 0.2

Assignment Tasks:

i. Calculate the Expected Value for Project A.
ii. Calculate the Expected Value for Project B.
iii. Based on the Expected Values, recommend which project the company should choose.

Answers

i. EV_A=230,000
ii. EV_B=235,000
iii. **Recommendation:** The company should choose Project B, as it has a higher Expected Value of $235,000 compared to Project A's $230,000. This indicates that, on average, Project B is expected to yield a higher profit.

Activity 3

VIDEO 2: Decision trees

1. **You are required to watch the Video on decision tree:**

The video provides an overview of decision trees and demonstrates how they can be used to make decisions when there is uncertainty about future outcomes.

Click here to watch the video: https://youtu.be/PYxDkGlpj_U?si=3OIlgOpXFcCbIss_

2. **Key Points to Focus on**
   - **Decision Trees Overview:** Understand that decision trees are tools used to make decisions in uncertain situations.
   - **Structure of Decision Trees:** Note that decision trees are composed of nodes and branches.
     - **Nodes:** Represent decisions or events.
     - **Branches:** Represent possible outcomes of those decisions or events.
   - **Expected Value Calculation:** Learn how to calculate the expected value of different decisions using decision trees.

- **Expected Value (EV):** The sum of the probabilities of each outcome multiplied by the value of that outcome.

- **Best Decision:** The decision with the highest expected value is considered the best choice.

3. **Take short Notes**
   - Summarize the key points from the video in your own words.
   - Include the structure of decision trees, how to calculate expected values, and the decision-making process.
4. Undertake **task 3** Masterly Exercise

Task 3

MASTERY EXERCISE 1.1

1. A company needs to decide between investing in a new technology and expanding its current facilities. Use the following details to create a decision tree:

- **Investment in New Technology:**
  - High Return: $500,000 with a probability of 0.3
  - Moderate Return: $200,000 with a probability of 0.5
  - Low Return: $50,000 with a probability of 0.2
- **Expansion of Current Facilities:**
  - High Return: $400,000 with a probability of 0.4
  - Moderate Return: $150,000 with a probability of 0.4
  - Low Return: $30,000 with a probability of 0.2

Required:

1. Draw the decision tree for both investment options.
2. Calculate the expected value for each option.
3. Determine which option has the highest expected value and explain why it is the best choice.

Task 4

READING MATERIAL 2: BOOK CHAPTER

To enhance your understanding of decision theory concepts and methodologies read Chapter 15 of Hillier and Lieberman's textbook: Hillier, F. S., & Lieberman, G. J. (2015). *Introduction to Operations Research*. McGraw-Hill.

Chapter: 15 – Decision Theory

Pages: 749–769

1. **Focus Areas:**
    - **Decision Making under Uncertainty:** Understand the fundamental concepts of decision-making in uncertain environments.
    - **Decision Trees:** Learn about the construction and analysis of decision trees for making informed choices.
    - **Decision Criteria:** Review the different criteria for decision-making, including expected value and other decision rules discussed in the chapter.
    - **Examples and Applications:** Pay attention to the examples provided in the chapter to see how decision theory is applied in real-world scenarios.

Task 5

END OF MODULE ASSESSMENT

Question 1: Maximax Criterion Application

A company is deciding between three investment options based on their potential payoffs under different economic conditions. The payoff matrix is as follows:

Investment/Condition	Boom Economy	Stable Economy	Recession Economy
A	$120,000	$80,000	$40,000
B	$100,000	$90,000	$30,000
C	$110,000	$70,000	$20,000

Using the Maximax Criterion, which investment should the company choose, and why?

Answer:

Using the Maximax Criterion, the company should choose Investment A.

Question 2: Maximin Criterion Application

A business is evaluating two projects based on their payoffs under various market conditions. The payoff matrix is:

Project/Condition	High Demand	Medium Demand	Low Demand
X	$250,000	$150,000	$50,000
Y	$200,000	$180,000	$70,000

Using the Maximin Criterion, which project should the business choose, and why?

Answer:

Using the Maximin Criterion, the business should choose Project Y.

Question 3: Minimax Regret Criterion Application

Consider a firm evaluating three strategies based on the following payoff matrix:

Strategy/Condition	High Success	Moderate Success	Low Success
1	$300,000	$200,000	$50,000
2	$250,000	$220,000	$40,000
3	$270,000	$210,000	$60,000

Calculate the regrets and determine which strategy to choose using the Minimax Regret Criterion.

Answer:

To use the Minimax Regret Criterion:

1. Determine the best payoff for each condition:
   - High Success: $300,000 (Strategy 1)
   - Moderate Success: $220,000 (Strategy 2)
   - Low Success: $60,000 (Strategy 3)
2. Calculate the regret matrix:

Strategy/Condition	High Success	Moderate Success	Low Success
1	$0	$20,000	$10,000
2	$50,000	$0	$20,000
3	$30,000	$10,000	$0

3. Identify the maximum regret for each strategy:
   - Strategy 1: $20,000
   - Strategy 2: $50,000
   - Strategy 3: $30,000
4. The strategy with the minimum of the maximum regrets is Strategy 1, with a maximum regret of $20,000.

Question 4: Expected Value Calculation

A firm is considering a new product launch with the following possible outcomes:

- High Demand: Profit of $600,000 with a probability of 0.4
- Moderate Demand: Profit of $250,000 with a probability of 0.3
- Low Demand: Loss of $100,000 with a probability of 0.3

Calculate the Expected Value (EV) of the product launch.

Answer:

The Expected Value of the product launch is $285,000.

Question 5: Decision Trees Analysis

A company is using a decision tree to evaluate whether to invest in a new technology. The decision tree includes two branches for the market conditions: favorable and unfavorable. The probabilities and payoffs are as follows:

- Favorable Market (Probability = 0.6): Profit of $500,000
- Unfavorable Market (Probability = 0.4): Loss of $200,000

Calculate the Expected Value of investing in the new technology.

Answer:

The Expected Value of investing in the new technology is $220,000.

REFERENCE LIST AND FURTHER READING

1. Decisionary. (2023, May 27). *Decision-making under uncertainty*. YouTube. https://www.youtube.com/watch?v=XYtnnOjLEno
2. Hillier, F. S., & Lieberman, G. J. (2015). *Introduction to operations research*. McGraw-Hill.
3. Marshall, K. T., and R. M. Oliver: Decision Making and Forecasting, McGraw-Hill, New York,1995.
4. Steland, A. (2012). *Financial statistics and mathematical finance: methods, models and applications*: John Wiley & Sons.
5. tutor2u. (2017). Decision Trees for A Level Business. In *YouTube*. https://www.youtube.com/watch?v=PYxDkGlpj_U

TAKE HOME MESSAGE

Now that you've navigated through this module, think about the key insights you've gathered. If you were to summarize your biggest takeaway in one powerful statement, what would it be?

How will you apply what you've learned in decision-making when faced with uncertainty? Reflect on how this knowledge changes the way you approach challenges, and craft your own take-home message; one that will guide your future decisions.

WHAT'S NEXT?

- Module 8: Review Assignment
- Module 9: Statistics for Business
- As we move forward, the next module, Statistics for Business, will equip you with the tools to make data-driven decisions. In today's business environment, understanding how to collect, analyze, and interpret data is key to gaining a competitive edge. Prepare to dive into the world of descriptive statistics, measures of central tendency, and dispersion, which will form the foundation for effective decision-making and problem-solving. Let's explore how numbers can tell the story behind business success!

MODULE 9: STATISTICS FOR BUSINESS

INSTRUCTIONAL HOURS: 4

MODULE OVERVIEW

In this module, you will explore the fundamental concepts of statistics as they apply to business analysis. We will begin with an introduction to descriptive statistics, which are essential for summarizing and interpreting data. You will learn about measures of central tendency, such as mean, median, and mode, which provide insights into the typical values within a dataset. The module will also cover measures of dispersion, including range, variance, and standard deviation, which help you understand the spread and variability of data. By the end of this module, you will have the skills to effectively summarize and analyze business data, enabling you to make informed decisions based on statistical insights.

LEARNING OUTCOMES

By the end of this module, you should be able to:

1. Describe key concepts in descriptive statistics.
2. Demonstrate proficiency in computing measures of central tendency and measures of dispersion for business datasets.
3. Apply statistical techniques to summarize, analyze and interpret business data effectively.
4. Evaluate the role of descriptive statistics in business decision-making.

LEARNING ACTIVITIES/TASK LIST

1. Review power presentation PDF
2. Participate in an online discussion
3. Reading material 1
4. Complete participation assignment: Quiz 1
5. Watch video
6. Reading Material 2: Lecture notes
7. Participate in group discussion
8. Complete the mastery exercise.
9. Complete the end-of-module assessment.

INSTRUCTIONAL MATERIALS & LEARNING ACTIVITIES

(Learners to engage in self-directed learning

READING MATERIAL 1
(Notes, presentation, an article, a book chapter, case study, video, please add assessment…)

Activity 1
Review the Notes and upon completion; undertake group discussion (Task 2)

INTRODUCTION TO BUSINESS STATISTICS

To begin this session, consider Sweet Aroma, a small local coffee shop, wants to improve its business but doesn't know where to start. They've collected data on daily sales, customer preferences, and foot traffic for the past three months. However, the manager is unsure how to use this information.

Using **basic statistics**, Sweet Aroma can:

1. Calculate the **average daily sales** to determine trends and busy periods.
2. Identify the most popular items through **frequency distribution**.
3. Understand customer preferences by analyzing the **percentage** of each type of coffee sold.

This simple analysis helps Sweet Aroma adjust its offerings, improve marketing, and manage staff schedules more effectively.

This scenario shows the **practical need for statistics** in everyday business operations.

Definition of statistics

Statistics is the science of collecting, analyzing, interpreting, and presenting data to make informed decisions. It involves the use of mathematical methods to summarize and derive meaningful insights from data sets, helping to identify patterns, relationships, and trends.

Illustration 1

Imagine you are a teacher who wants to understand the performance of your students in a recent math test. You collect data on the scores of all the students. Here's how statistics would come into play:

1. **Collecting Data:** You gather the test scores of all 30 students in your class.
2. **Organizing Data:** You create a list of the scores and group them into ranges, like 50-60, 61-70, and so on.
3. **Analyzing Data:** You calculate various statistical measures:
    - **Mean (Average):** Add up all the scores and divide by the number of students to find the average score.
    - **Median:** Find the middle score when all the scores are arranged in ascending order.
    - **Mode:** Identify the score that appears most frequently.
    - **Standard Deviation:** Measure the amount of variation or dispersion in the scores.
4. **Interpreting Data:** Based on the mean, median, and standard deviation, you might conclude that most students performed well, but there was a wide range of scores, indicating some students struggled more than others.
5. **Presenting Data:** You create a graph, such as a histogram or a box plot, to visually represent the distribution of scores. This helps you easily see patterns, such as whether most students scored in a particular range or if there are any outliers.
6. **Organizing Data for Decision Making:** You use the information to decide if any additional help is needed for students who scored lower, or if the test was too difficult.

In summary, statistics helps in summarizing data, finding patterns, and making decisions based on empirical evidence.

Application of statistics in business

In today's data-driven world, businesses are increasingly relying on statistical methods to navigate complex decisions and drive strategic growth. Statistics, the science of collecting, analyzing, interpreting, and presenting data, plays a pivotal role in various aspects of business management.

By transforming raw data into meaningful insights, statistics enables businesses to make informed decisions, optimize processes, and enhance their competitive edge. Some of the areas include:

1. Market Research

Businesses use statistics to analyze market trends, understand consumer preferences, and identify potential customer segments. By analyzing survey data and sales figures, companies can make strategic decisions about product development, pricing, and marketing strategies.

Example: A company might survey customers to find out their preferred features in a new product. Statistical analysis of the survey responses can reveal the most desired features, guiding the product design.

2. Financial Analysis

Statistics helps businesses assess financial performance through metrics such as revenue, profit margins, and return on investment (ROI). It is also used in forecasting financial trends and budgeting.

Example: A company might use statistical methods to analyze historical sales data and forecast future revenue. This can help in setting realistic sales targets and preparing financial budgets.

3. Quality Control

In manufacturing and service industries, statistical techniques are used to monitor and improve quality. Control charts, hypothesis testing, and regression analysis are used to detect defects, analyze variations, and implement quality improvements.

Example: A factory might use statistical process control (SPC) charts to monitor production quality. If defects exceed a certain threshold, the data helps identify and address the root causes.

4. Risk Management

Statistics helps businesses assess and manage risks by analyzing potential uncertainties and their impacts. Techniques such as probability analysis and risk modeling are used to evaluate and mitigate risks.

Example: An insurance company uses statistical models to predict the likelihood of claims and set appropriate premiums. Statistical analysis of past claims data helps in estimating future risks.

5. Human Resources

In HR, statistics is used for analyzing employee performance, recruitment effectiveness, and employee satisfaction. This helps in making decisions related to hiring, promotions, and training programs.

Example: A company might use statistical analysis to evaluate the effectiveness of different recruitment channels by comparing the performance and retention rates of employees hired through each channel.

6. Sales and Marketing

Statistics helps in analyzing sales performance, customer behavior, and marketing campaign effectiveness. Businesses use data analysis to optimize marketing strategies and improve sales outcomes.

Example: A retailer might analyze sales data to determine which products are most popular in different seasons. This helps in planning inventory and marketing strategies for upcoming periods.

7. Operations Management

Statistics is used to optimize operations by analyzing processes, identifying inefficiencies, and improving workflows. Techniques such as queuing theory and optimization models are applied to enhance operational efficiency.

Example: A logistics company might use statistical models to optimize delivery routes, reducing transportation costs and improving delivery times.

In summary, statistics provides businesses with valuable insights into their operations, market conditions, and financial performance. By applying statistical methods, companies can make data-driven decisions, enhance efficiency, and achieve better outcome

Task 1

ONLINE DISCUSSION

Use chats, forums, journals, wikis, blogs, question/answer, message my teacher to engage others.

Consider a company that has recently launched a new product. The marketing team wants to understand the effectiveness of their advertising campaign and assess customer satisfaction with the new product. How can statistical methods be used to analyze the data collected from customer feedback and sales figures to make informed decisions about the product and future marketing strategies?

In your response, discuss the types of data that would be collected, the statistical techniques that could be applied to analyze this data, and how these analyses can inform business decisions.

Post your answers in the LMS and engage with your peers.

Descriptive Statistics

In this section, you will acquire the skills necessary to effectively summarize and present data using measures of central tendency and dispersion, as well as how to select the most appropriate measures for summarizing business data.

Activity 2

READING MATERIAL 1: BOOK CHAPTER

Please read **Chapter 2:** Descriptive Statistics: Tabular and Graphical Presentations (pages 35-51) in the textbook by David, R. A., Dennis, J. S., Thomas, A. W., Jeffrey, D. C., James, J. C., Michael, J. F., & Jeffrey, W. O. (2019), Essentials of Statistics for Business and Economics.

This section introduces the essential methods for organizing and presenting data through tables and graphs. Pay close attention to how different types of data (e.g., categorical, quantitative) are represented using bar charts, pie charts, histograms, and frequency distributions. You will also learn about the key differences between these presentations and how they can be effectively used in a business context.

After reading, be prepared to discuss:

1. The importance of visual data presentation in business decision-making.
2. The benefits of using tables versus graphs in different scenarios.
3. Attempt questions/Quiz labeled **task 2**

Task 2

QUIZ/ QUESTIONS

1. A company collected data on the number of units sold in different regions during a specific quarter. The sales data (in units) from 10 regions is as follows:

Region	Units Sold
A	120
B	90
C	150
D	80
E	100
F	130
G	140
H	110
I	85
J	95

i. Create a frequency distribution table showing the number of regions that sold units in intervals of 80-99, 100-119, 120-139, and 140-159.

ii. Calculate the relative frequency for each interval.

Solution

Units Sold (Interval)	Frequency	Relative Frequency
80-99	4	4/10 = 0.40
100-119	3	3/10 = 0.30

120-139	**2**	**2/10 = 0.20**
140-159	**1**	**1/10 = 0.10**

2. Given the following test scores of students in a business course:
   65,70,75,80,85,90,95,70,85,75,80,65,70,75,95,80,85,70,75,85

   Required

   i. Organize the test scores into a frequency distribution using appropriate intervals
   ii. Draw a histogram based on the frequency distribution.

   Solution

Test Score Interval	Frequency
60-69	2
70-79	7
80-89	6
90-99	2

In the histogram:

- X-axis represents the score intervals (60-69, 70-79, etc.).
- Y-axis represents the frequency (2, 7, 6, 2).

Activity 3

VIDEO 2: Introduction to Measures of Central Tendency

Please watch the video, which covers the **Measures of Central Tendency**. **Central Tendency** refers to statistical measures that identify a single value that best represents the center point or typical value of a dataset. The most commonly used measures of central tendency are the **mean**, **median**, and **mode**. These measures summarize the data by pinpointing a central value, enabling businesses and analysts to grasp the "average" behavior or key characteristics of the dataset. By understanding these measures, decision-makers can gain insights into patterns, trends, and typical outcomes within their data, leading to more informed business decisions.

Click here to watch the video:
https://youtu.be/zC6e47l0tV4?si=nAOz8u3gxDEhgkZr

The video introduces three key measures of central tendency: the **mean**, **median**, and **mode**.

The video also explains the **advantages and disadvantages** of each measure:

After watching the video, summarize its content, focusing on:

1. Definitions of the mean, median, and mode.
2. The pros and cons of each measure of central tendency.
3. How skewness affects each measure.

Be prepared to discuss how these measures can be applied in business decision-making.

Activity 4

READING MATERIAL 2: LECTURE NOTES

1. MEAN

The **mean** is one of the most widely used measures of central tendency. It represents the average value of a data set and provides insight into the general tendency of the values.

The Mean for Non-Grouped Data

For non-grouped data, the **mean** is calculated by summing all the data points and dividing by the total number of data points. It is commonly referred to as the **arithmetic mean**.

The arithmetic mean is given by; $Mean\ (\mu) = \frac{\sum X}{n}$

Where; $\sum X$= sum of all data points
n = the number of data points

Example

Consider the following set of test scores for a group of students: 65,70,75,80,85

To find the mean:

Add the values: 65+70+75+80+85=375

Divide by the number of values: 375÷5=75

Thus, the mean score is 75.

The Mean for Grouped Data

When dealing with **grouped data**, the data is organized into classes, and the mean is calculated using the **midpoint** (or class mark) of each class. The midpoint of a class is the average of the lower and upper boundaries of that class.

Formula: $Mean\ (\mu) = \frac{\sum(f \times X_m)}{N}$

Where:

- f = frequency of each class
- Xm = midpoint of each class
- N = total number of observations

Steps to Calculate the Mean for Grouped Data:

1. **Determine the midpoints (class marks)** for each class.

$$\text{Midpoint Xm} = \frac{Lower\ Limit + Upper\ Limit}{2}$$

2. **Multiply the midpoint by the class frequency** to get f·Xm
3. **Sum the products of frequency and midpoints**.
4. **Divide the sum by the total frequency** N.

Example:
Consider the following frequency distribution of students' test scores:

Class Interval	Frequency (f)
60-69	3
70-79	6
80-89	5
90-99	2

Step 1: Calculate the midpoints for each class.

$$\text{Xm} = \frac{Lower\ Limit + Upper\ Limit}{2}$$

For the class 60-69, the midpoint is:

$$\text{Xm} = \frac{60 + 69}{2} = 64.5$$

For the class 70-79, the midpoint is:

$$\text{Xm} = \frac{70 + 79}{2} = 74.5$$

For the class 80-89, the midpoint is:

$$\text{Xm} = \frac{80 + 89}{2} = 84.5$$

For the class 90-99, the midpoint is:

$$\text{Xm} = \frac{90 + 99}{2} = 8 = 94.5$$

Step 2: Multiply the midpoints by the respective frequencies.

f·Xm=Frequency×Midpoint

Class Interval	Frequency (f)	Midpoint(Xm)	f·Xm
60-69	3	64.5	3×64.5=193.5
70-79	6	74.5	6×74.5=447.0
80-89	5	84.5	5×84.5=422.5
90-99	2	94.5	2×94.5=189.0

Step 3: Sum the products of frequency and midpoints.
193.5+447.0+422.5+189.0=1252.0

Step 4: Calculate the mean by dividing the sum by the total frequency N.
N=3+6+5+2=16

$$Mean\ (\mu) = \frac{1252.0}{16} = 78.25$$

Thus, the mean score is **78.25**.

Characteristics of the Mean

- Sensitive to extreme values (outliers): The mean is influenced by every data point, making it sensitive to unusually high or low values.
- Applicable to interval and ratio data: The mean can only be used for quantitative data.
- Unique: Every data set has a single, unique mean.

Advantages and Disadvantages of the Mean

Advantages:

1. Easy to understand and compute: For small data sets, the mean is simple to calculate and interpret.
2. Uses all data points: The mean considers every data point in the data set, providing a comprehensive measure of central tendency.
3. Suitable for further analysis: The mean is often used in advanced statistical techniques, such as regression analysis and hypothesis testing.

Disadvantages:

1. Affected by outliers: Extreme values (outliers) can significantly distort the mean, making it less representative of the data.
2. Not appropriate for skewed distributions: In highly skewed data, the mean may not accurately reflect the central tendency, as it is pulled toward the skew.
3. Not suitable for categorical data: The mean is only valid for numerical data and cannot be used for nominal or ordinal variables.

Exercise

Imagine you are analyzing the salaries (in \$1000s) of employees in a company:

Salary Range	Frequency (f)
40-49	5
50-59	8
60-69	12
70-79	3

Using the steps shown earlier for grouped data, calculate the mean salary by determining the midpoints and applying the formula.

Median

The **median** is a measure of central tendency that represents the middle value of a dataset when it is arranged in ascending or descending order. Unlike the mean, the median is not influenced by outliers, making it a better measure of central tendency for skewed distributions.

The Median for Non-Grouped Data

For **non-grouped data**, the median is simply the middle value of a sorted data set. If the number of data points is **odd**, the median is the middle value. If the number of data points is **even**, the median is the average of the two middle values.

Steps to Find the Median for Non-Grouped Data:

1. Arrange the data points in ascending or descending order.
2. If the number of data points n is **odd**, the median is the value at the position:

$$\text{Median Position} = \frac{n+1}{2}$$

3. If n is **even**, the median is the average of the two middle values at positions:

$$\text{Median Position} = \frac{n}{2} \; and \; (\frac{n}{2} + 1)$$

Example

Consider the following test scores: 56,70,65,80,90

Step 1: Sort the data: 56,65,70,80,90

Step 2: Since there are 5 values (an odd number), the median is the value in the middle position:

$$\text{Median Position} = \frac{5+1}{2} = 3rd\ value = 70$$

Example with an Even Number of Data Points:

Consider the data: 20,35,40,50,55,60

Step 1: Sort the data: 20,35,40,50,55,60

Step 2: Since there are 6 values (an even number), the median is the average of the two middle values (the 3rd and 4th values):

$$\text{Median Position} = \frac{40+50}{2} = 45$$

The Median for Grouped Data

When dealing with **grouped data**, the median is calculated using class intervals. The class that contains the median is referred to as the **median class**. The following formula is used to calculate the median for grouped data:

$$\text{Median } = \text{L} + (\frac{\frac{N}{2} - F}{f_m}) \times h$$

Where;

L = lower boundary of the median class

N = total frequency

F = cumulative frequency of the class before the median class

f_m = frequency of the median class

h= class width

Steps to Calculate the Median for Grouped Data:

1. **Find the median class**: The class interval where $\frac{N}{2}$ lies.
2. Use the formula to compute the median.

Example

Consider the following frequency distribution of scores:

Class Interval	Frequency (f)	Cumulative Frequency (CF)
40-49	5	5
50-59	8	13
60-69	12	25
70-79	3	28

Step 1: Determine the total number of data points N:
N=5+8+12+3=28

Step 2: Find $\frac{N}{2}$

$$\frac{28}{2} = 14$$

The 14th value lies in the class interval 60-69, making this the **median class**.

Step 3: Apply the formula:

$$\text{Median } = \text{L} + \left(\frac{\frac{N}{2} - F}{f_m}\right) \times h$$

L=60 (lower boundary of the median class)
F=13 (cumulative frequency of the class before the median class)
f_m=12 (frequency of the median class)
h=10 (class width)

$$\text{Median } = 60 + \left(\frac{14 - 13}{12}\right) \times 10 = 60.83$$

Thus, the median is approximately **60.83**.

After the task, you are to undertake group discussion labeled **task 4.**

Advantages and Disadvantages of the Median

Advantages

1. Not affected by outliers or skewed data: The median remains unchanged even if there are extreme values in the dataset, making it a more robust measure of central tendency for skewed distributions.
2. Easy to calculate for small data sets: For small data sets, the median can be quickly determined.
3. Can be used with ordinal data: Unlike the mean, the median can be applied to ordinal data where numerical values are not available but ranks are given.

Disadvantages

1. Not as mathematically tractable as the mean: The median is not commonly used in advanced statistical methods, such as regression, where the mean is preferred.
2. Does not use all data points: Unlike the mean, the median only considers the middle position, ignoring the rest of the data points.
3. Less efficient for large data sets: Sorting data to find the median can be time-consuming for large data sets.

Application of the Median in Business

The median is widely used in business when data distributions are skewed or have outliers. Examples include:

i. Income Distribution: The median is often used to report average income because income distributions are typically skewed, with a few high-income earners raising the mean.

ii. Real Estate: In real estate, the median home price is often reported instead of the mean because outlier properties with extremely high prices can distort the average.

iii. Customer Feedback: The median score in customer satisfaction surveys helps businesses understand typical feedback, especially when a few extreme ratings might skew the mean.

Exercise

Consider a frequency distribution of exam scores:

Class Interval	Frequency
40-49	3
50-59	6
60-69	9
70-79	4

Required

Find the median

Mode

The **mode** is the value or values that occur most frequently in a dataset. Unlike the mean and median, the mode is applicable to both numerical and categorical data. The mode is particularly useful when analyzing data with repeated values and is commonly used in business to identify the most common or popular items, prices, or responses in a dataset.

The Mode for Non-Grouped Data

For **non-grouped data**, the mode is the data point that appears most frequently. A dataset may have:

- **One mode** (unimodal),

- **Two modes** (bimodal), or
- **More than two modes** (multimodal).

Steps to Find the Mode for Non-Grouped Data:

1. List all the data points.
2. Identify the value(s) that appear most frequently.

Example:

Consider the following dataset representing shoe sizes: 7,8,9,7,10,7,9,8,8

Step 1: Count the frequency of each value:

- 7 appears 3 times.
- 8 appears 3 times.
- 9 appears 2 times.
- 10 appears 1 time.

Step 2: Identify the mode: Since both 7 and 8 appear 3 times, the dataset is **bimodal**, with the modes being **7** and **8**.

The Mode for Grouped Data

For **grouped data**, the mode is found by identifying the **modal class**; the class interval with the highest frequency. Once the modal class is identified, the mode can be estimated using the following formula:

$$\text{Mode } = \mathrm{L} + \left(\frac{f_m - f_1}{(\boldsymbol{fm} - \boldsymbol{f1}) + (\boldsymbol{fm} - \boldsymbol{f2})}\right) \times h$$

Where:

L= lower boundary of the modal class
fm = frequency of the modal class
f_1 = frequency of the class preceding the modal class
f_2 = frequency of the class succeeding the modal class
h= class width

Steps to Calculate the Mode for Grouped Data:

1. Identify the modal class (the class with the highest frequency).
2. Apply the formula to estimate the mode.

Example

Consider the following grouped data on employee salaries:

Salary Range	Frequency (f)
20,000-30,000	5
30,000-40,000	8
40,000-50,000	12
50,000-60,000	10

Step 1: Identify the modal class: The class interval **40,000-50,000** has the highest frequency (12), so it is the **modal class**.
Step 2: Apply the formula:
- $L=40,000$ (lower boundary of the modal class),
- $f_m=12$,
- $f_1=8$ (frequency of the class before the modal class),
- $f_2=10$ (frequency of the class after the modal class),
- $h=10,000$ (class width).

$$\text{Mode } = 40{,}000 + \left(\frac{12-8}{(12-8)+(12-10)}\right) \times 10{,}000$$
$$\text{Mode } = 40{,}000 + \left(\frac{4}{(4+2)}\right) \times 10{,}000$$
$$\text{Mode } = 40{,}000 + \left(\frac{4}{6}\right) \times 10{,}000 = 40{,}000 + 6{,}666.67 = 46{,}666.67$$

Thus, the mode is approximately **46,666.67**.
Characteristics of the Mode
- **Can be used with any type of data**: The mode can be calculated for **nominal, ordinal, interval**, and **ratio** data, making it versatile.
- **More than one mode possible**: A dataset can be **unimodal**, **bimodal**, or **multimodal**, depending on the number of peaks or repeated values.
- **May not exist**: In some datasets, no value repeats, and thus no mode exists.
- **Insensitive to outliers**: Like the median, the mode is not influenced by extreme values in a dataset.

Advantages and Disadvantages of the Mode
Advantages:
1. **Applicable to categorical data**: The mode is the only measure of central tendency that can be used with **categorical (nominal)** data, such as product preferences or survey responses.
2. **Easy to understand and calculate**: The mode is simple to identify in small datasets.
3. **Not affected by extreme values**: Since the mode only considers the most frequent value(s), outliers do not affect it.

Disadvantages:
1. **Not unique**: A dataset can have more than one mode (bimodal or multimodal), making it less informative in some cases.
2. **Not useful for small datasets**: In very small datasets, the mode may not represent a meaningful central value.

3. **May not exist**: If no value repeats, the mode does not exist, which can limit its application.

Application of the Mode in Business

The mode is especially useful in business when identifying the most common category, value, or outcome. It can be applied in various fields, including:

i. Retail and Product Development: Companies use the mode to determine the most popular product sizes, colors, or models based on customer preferences.
ii. Marketing: The mode helps in identifying the most frequent responses in customer feedback, surveys, and polls, helping businesses focus on the most common customer concerns or preferences.
iii. Inventory Management: Businesses can track the most frequently sold items to ensure that the mode (most popular product) is well-stocked.

Exercise

Consider a survey of daily travel times, summarized in a frequency distribution:

Travel Time (minutes)	Frequency
10-20	5
20-30	7
30-40	12
40-50	8

Required

Estimate the most common travel time.

Measures of Dispersion

1. Range

The range is the simplest measure of variability. It is the difference between the highest and lowest values in a dataset.

Formula:

Range=Maximum value−Minimum value

Range=Maximum value−Minimum value

Example:

Consider a dataset of daily phone usage (in minutes) for a group of college students:

130,160,195,210,230

Maximum value = 230 minutes
Minimum value = 130 minutes

Range=230−130=100 minutes

Thus, the range of phone usage is 100 minutes.

Limitations of the Range

The range only considers the two extreme values, which may not provide an accurate representation of variability if the dataset contains outliers.

2. **Interquartile Range (IQR)**

The interquartile range (IQR) measures the spread of the middle 50% of the data, making it less sensitive to outliers. It is the difference between the third quartile (Q3) and the first quartile (Q1), where:

Q1 represents the 25th percentile of the data.
Q3 represents the 75th percentile of the data.

Formula:
IQR=Q3−Q1
Example:

Using the following dataset of phone usage (in minutes) from college students:
130,160,195,210,230

Step 1: Arrange the data in ascending order (already done).
Step 2: Calculate Q1 and Q3:
Q1 (25th percentile) = 160 minutes
Q3 (75th percentile) = 210 minutes

IQR=210−160=50 minutes

Thus, the IQR is 50 minutes, indicating that the middle 50% of phone usage data spans 50 minutes.

3. Variance

The **variance** measures the average squared deviation from the mean. It provides a comprehensive measure of variability by considering how far each data point lies from the mean.

Formula:

$$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

Where:

- x_i = individual data point,
- $\bar{x}$ = sample mean,
- n = number of data points.

Example

Consider the following dataset representing the daily phone usage (in minutes) for a group of high school students:

120,150,190,200,220

Step 1: Calculate the mean:

x¯=120+150+190+200+2205=176 minutes

x¯=5120+150+190+200+220=176 minutes

Step 2: Subtract the mean from each data point and square the result:

$(120-176)^2=3136$, $(150-176)^2=676$, $(190-176)^2=196$, $(200-176)^2=576$, $(220-176)^2=1936$

Step 3: Sum the squared deviations:

3136+676+196+576+1936=6520

Step 4: Divide by n−1 (degrees of freedom):

$$\mathcal{S}^2 = \frac{\mathbf{6520}}{\mathbf{5-1}} = \frac{\mathbf{6520}}{\mathbf{4}} = \mathbf{1630}$$

Thus, the variance is **1630** minutes squared.

Limitations of Variance

- The units of variance are squared, which can make interpretation difficult. This is why we often use the standard deviation, which has the same units as the data.

Standard Deviation

The standard deviation is the square root of the variance. It is the most widely used measure of variability because it provides an easy-to-interpret measure of spread in the same units as the original data.

Formula:

$$\mathcal{S} = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} \qquad =\sqrt{\mathcal{S}^2}$$

Example

Using the variance calculated above ($s^2=1630$), the standard deviation is:

s=$\sqrt{\mathbf{1630}}$ = 40.37 minutes

Thus, the standard deviation is approximately **40.37 minutes**.

Interpretation

- The standard deviation tells us that, on average, the daily phone usage of the high school students varies by **40.37 minutes** from the mean of 176 minutes.

Applications of Variability in Business

Measures of variability are crucial in business to understand the risk, uncertainty, and consistency of various outcomes. For example:

i. Sales Analysis: A company may analyze sales data variability to determine whether certain products have consistent sales across months or exhibit high fluctuations.
ii. Customer Behavior: Understanding variability in customer purchase patterns helps businesses tailor strategies to target more predictable (low variability) or unpredictable (high variability) customer segments.

Example

A retail store tracks daily sales over two weeks. Both Store A and Store B have average daily sales of $5,000, but Store A has a higher standard deviation of $800 compared to Store B's standard deviation of $300. This suggests that Store A's sales are more volatile, while Store B has more consistent sales.

Standard Deviation for Grouped Data

To calculate the standard deviation for grouped data, follow these steps:

1. Organize the Data

Assume you have a frequency distribution table with the following columns:

- **Class Interval**: The ranges of values (e.g., 10-20, 20-30).
- **Frequency (f)**: The number of occurrences in each interval.
- **Midpoint** ($\bar{x}$): The midpoint of each class interval, calculated as (Lower Bound+Upper Bound)/2

2. Calculate the Mean

i. First, find the mean (average) of the grouped data:
ii. Find the midpoint for each class interval:
iii. $\text{Midpoint} = \frac{Lower\ Bound + Upper\ Bound}{2}$
iv. Calculate the weighted mean:

$$Mean\ (\bar{x}) = \frac{\sum(f.\bar{x})^2}{\sum f}$$

Where; f is the frequency and $\bar{x}$ is the midpoint.

3. Calculate the Variance

Compute $(x_i - \bar{x})^2$ for each class interval:

Subtract the mean from each midpoint, square the result, and multiply by the class frequency:

$$f.(x_i - \bar{x})^2$$

Find the sum of these values:

$$Sum = \sum f.(x_i - \bar{x})^2$$

Calculate the variance:

$$Variance\ (\sigma^2) = \frac{Sum}{\sum f}$$

4. Calculate the Standard Deviation

Take the square root of the variance to find the standard deviation:

$$Standard\ Deviation(\sigma) = \sqrt{Variance}$$

$$Standard\ Deviation(\sigma) = \sqrt{\frac{\sum f.(x_i - \bar{x})^2}{\sum f}}$$

Example

Consider the following grouped data representing the test scores of students:

Class Interval	**f_i**	**x_i**	**$x_i - \bar{x}$**	**$(x_i - \bar{x})^2$**	**$f_i(x_i - \bar{x})^2$**
10-20	3	15	-26.25	689.06	2067.19
20-30	7	25	-16.25	264.06	1848.42
30-40	15	35	-6.25	39.06	585.94
40-50	10	45	3.75	14.06	140.63
50-60	5	55	13.75	189.06	945.31
Total	**40**				**5587.5**

- **Step 1: Determine the Class Midpoints:**

Midpoints (xi): 15, 25, 35, 45, 55

Step 2: Calculate the Mean:

$$Mean\ (\bar{x}) = \frac{3 \times 15 + 7 \times 25 + 15 \times 35 + 10 \times 45 + 5 \times 55}{3 + 7 + 15 + 10 + 5} = \frac{1650}{40} = 41.25$$

Step 3: Compute the Deviation for Each Class:

Deviations $(\boldsymbol{x_i} - \bar{\boldsymbol{x}})$: −26.25, −16.25, −6.25, 3.75, 13.75

Step 4: Square the Deviations:

Squared Deviations: $(\boldsymbol{x_i} - \bar{\boldsymbol{x}})^2$: 689.06, 264.06, 39.06, 14.06, 189.06

Step 5: Multiply by the Class Frequency:

$f.(x_i - \bar{x})^2$: 2067.19, 1848.42, 585.94, 140.63, 945.31

Step 6: Sum the Results and Divide by Total Frequency:

$$Sum = \sum f.(x_i - \bar{x})^2 = 5587.5$$

$$Variance\ (\sigma^2) = \frac{Sum}{\sum f} = \frac{5587.5}{40} = 139.69$$

Step 7: Take the Square Root to Find Standard Deviation:

$$Standard\ Deviation(\sigma) = \sqrt{Variance} = \sqrt{139.69} \approx\ 11.82$$

Thus, the standard deviation for this grouped data is approximately 11.82.

Interpretation

Given that the **mean** score is **41.25** and the **standard deviation** is **11.82**, the interpretation of the results becomes clearer:

1. **Spread Around the Mean**:
    - Most students' scores tend to cluster around the mean score of **41.25**.
    - With a standard deviation of **11.82**, the majority of the students scored within the range of **41.25 ± 11.82**, which is approximately **29.43** to **53.07**.
2. **Performance Interpretation**:
    - Scores within this range (29.43 to 53.07) are considered typical or within one standard deviation of the mean. Most students are likely to have scored in this range.

- If a student scored significantly above **53.07** or below **29.43**, their performance would be considered an outlier in either direction.

3. **Understanding Variability**:
    - The relatively high standard deviation compared to the mean suggests there is noticeable variability in the students' performance.
    - Some students performed much better or worse than the average score of **41.25**, indicating a wide range of abilities or achievement levels within the group.
4. **Performance Distribution**:
    - If the data follows a normal distribution, about **68%** of the students' scores would fall within **one standard deviation** from the mean, i.e., between **29.43** and **53.07**.
    - Approximately **95%** of the scores would fall within **two standard deviations** (between **17.61** and **64.89**).

Activity 5

ONLINE DISCUSSION

Use chats, forums, journals, wikis, blogs, question/answer, message my teacher to engage others.

In a recent exam, the class means score was 75, with a standard deviation of 8.5. What can you infer about the performance of students who scored more than one standard deviation above or below the mean? Discuss how the standard deviation provides insights into the consistency and variability of students' scores in relation to the average performance.

Discuss your response with your peers.

Task 3

MASTERY EXERCISE 1.1

1. Given the data set: 5, 8, 12, 15, 18, calculate the standard deviation.
2. Explain how outliers can affect the standard deviation of a data set.
3. Why the standard deviation considered a better measure of dispersion compared to the range?
4. Calculate the standard deviation for the following grouped data:

Class Interval	Frequency (f_i)
10 – 20	4
20 – 30	6
30 – 40	10
40 – 50	8
50 – 60	2

5. Find the interquartile range (IQR) for the following data set: 25, 28, 31, 34, 36, 39, 42, 45.
6. calculate the IQR from the grouped in question 4:

Answers

1. ≈4.67
2. Outliers can increase the standard deviation because they create larger deviations from the mean, which results in a higher variance and thus a higher standard deviation.
3. The standard deviation takes into account all values in the data set and their distance from the mean, while the range only considers the difference between the largest and smallest values, which can be misleading if there are outliers.
4. ≈10.91
5. QR = Q3 - Q1 = 40.5 - 29.5 = 11
6. Rubrics: Calculate cumulative frequencies and determine Q1 (25th percentile) and Q3 (75th percentile) using interpolation within the intervals.
   Q1 lies in the 20–30 interval, and Q3 lies in the 40–50 interval.
   Use the formula for interpolation to find exact values, then subtract Q1 from Q3 to find the IQR
   IQR=Q3−Q1=43.125−25.83=17.295
   Final Answer:
   The **Interquartile Range (IQR)** for the grouped data is approximately **17.30**.

Activity 6

END OF MODULE ASSESSMENT 1

1. Given the following data set:

12, 15, 19, 22, 15, 19, 24, 29, 15, 19, 22, 15

a) Calculate the mean of the data set.

b) Identify the median of the data set.

c) Determine the mode of the data set.

(Show your working steps)

2. The following table shows the frequency distribution of scores from a test:

Class Interval	Frequency (f)
10 – 20	3
20 – 30	7
30 – 40	12
40 – 50	10
50 – 60	5

Calculate the **mean** of the grouped data using the midpoints of the class intervals. *(Show your working steps)*

3. Consider the data set of the following scores:

5, 9, 12, 18, 22

a) Calculate the variance and standard deviation for this data set.

b) Interpret what the standard deviation indicates about the variability of the scores.

4. Given the data set:

10, 12, 15, 18, 20, 22, 25

a) Calculate the mean and standard deviation of the data.

b) Add an outlier value of 100 to the data set and recalculate the mean and standard deviation.

c) Explain how the outlier affects the mean and standard deviation of the data.

5. The ages of a group of employees at a company are as follows: **25, 28, 31, 35, 38, 42, 45, 50, 54, 58**

a) Find the **first quartile (Q1)** and **third quartile (Q3)** for the data set. **b)** Calculate the **interquartile range (IQR)** and explain what it indicates about the spread of the middle 50% of the data.

6. A group of students scored the following marks in a quiz: **55, 62, 69, 71, 75, 80, 82, 85**

a) Calculate the **range** of the data.

b) Calculate the **variance** and the **standard deviation** for the data set. **c)** Provide an interpretation of the spread and consistency of the students' scores based on the standard deviation.

Answers

1. a) ≈18.83
   b) 19
   c) 15
2. 36.89
3. a) Variance=13.448; b) Standard deviation≈3.67
4. a) Standard deviation: ≈5.07 b) New standard deviation: ≈29.26 c) The outlier increases both the mean and the standard deviation significantly, showing how it pulls the data away from the center and increases variability.
5. a)
   Q1 = median of the first half: 31
   Q3 = median of the second half: 50

   b)
   IQR = Q3−Q1=50−31=19Q3−Q1=50−31=19
   The IQR shows that the middle 50% of the data has a spread of 19 unit
6. a) Range=85−55=30, b) variance ≈112.98; standard deviation ≈10.63
   c) The standard deviation indicates moderate variability in students' scores, meaning there is some spread but not extreme.

REFERENCE LIST AND FURTHER READING

1. Anderson, D. R., Williams, T. A., & Cochran, J. J. (2020). Statistics for business & economics. Cengage Learning.
2. Daniel Storage (2019). *Measures of Central Tendency (Mean, Median, Mode)*. You tube. https://www.youtube.com/watch?v=zC6e47I0tV4
3. David, R. A., Dennis, J. S., Thomas, A. W., Jeffrey, D. C., James, J. C., Michael, J. F., & Jeffrey, W. O. (2019). Essentials of statistics for business and economics.

4. McClave, J. T., Benson, P. G., & Sincich, T. (2008). *Statistics for business and economics*. Pearson Education.

5. *Statistics for Business and Economics* (13th ed.). (2017). McClave, J. T., & Sincich, T. Pearson.

TAKE HOME MESSAGE

Understanding statistics is more than just crunching numbers; it's about interpreting data to reveal insights, make predictions, and drive smart business decisions. As you move forward, remember that every mean, variance, and trend you analyze is a step towards solving real-world problems. Let the skills you've gained in both Business Mathematics and Statistics for Business empower you to lead with confidence, turning data into action and challenges into opportunities

WHAT'S NEXT?

- Module 9 Review Assignment
- Reflection on the entire course and final overall course assessment

Congratulations on Completing the Business Mathematics and Statistics Course!

As you conclude this comprehensive course, we celebrate your achievement in mastering a diverse range of essential business mathematics concepts. Over the span of this journey, you have:

- Explored basic mathematical concepts and applied fundamental tools to various business scenarios.
- Gained proficiency in Basic Algebra and Arithmetic, including operations with real numbers, algebraic expressions, and linear equations, enhancing your ability to perform critical business analyses like break-even analysis.
- Delved into Functions and Graphs, learning to define and graph linear, quadratic, exponential, and logarithmic functions, and applying these functions to business scenarios such as cost, revenue, and profit analysis.
- Acquired skills in Matrix Algebra, understanding matrices, their operations, and their applications in business settings.
- Tackled Optimization Problems, mastering linear programming techniques, graphical solutions, and applications for resource allocation, profit maximization, and cost minimization.
- Studied the Mathematics of Finance, covering interest calculations, annuities, perpetuities, loan amortization, and investment appraisal to make informed financial decisions.
- Engaged with Calculus for Business, including limits, differentiation, and integration, and their practical applications in optimizing business functions and analyzing marginal changes.

- Applied Decision Analysis techniques to make informed choices under uncertainty, using expected value, decision trees, and risk analysis.
- Analyzed data with Statistics for Business, mastering descriptive statistics, measures of central tendency, and dispersion to interpret and utilize business data effectively.

Your dedication and hard work throughout this course have been truly commendable. You've developed a robust set of mathematical and statistical tools that will serve as a strong foundation for your future endeavors in the business world. Your ability to analyze data, solve complex problems, and make informed decisions has been significantly enhanced, positioning you for success in your professional journey.

Congratulations once again on your accomplishment!
Your perseverance and commitment have set you up for continued success, and we wish you all the best as you apply these skills in your future pursuits.